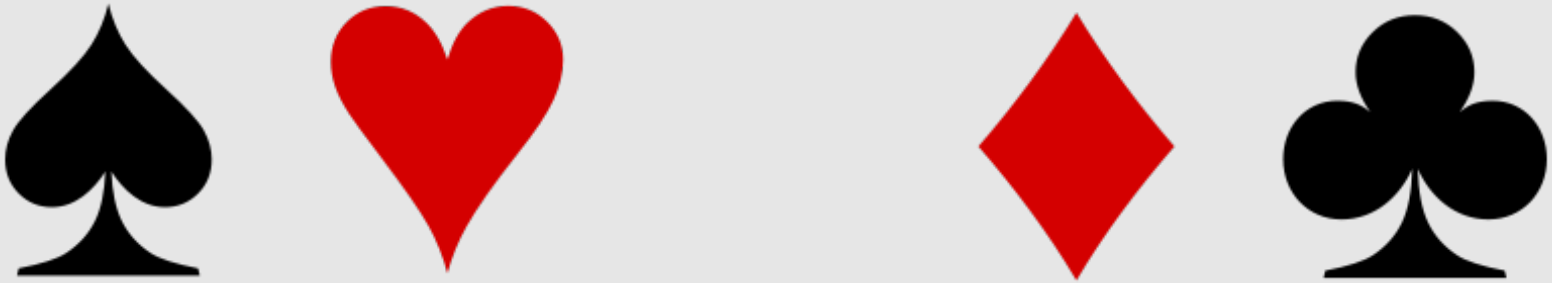
Monte-Carlo tree search for multi-player, no-limit Texas hold'em poker

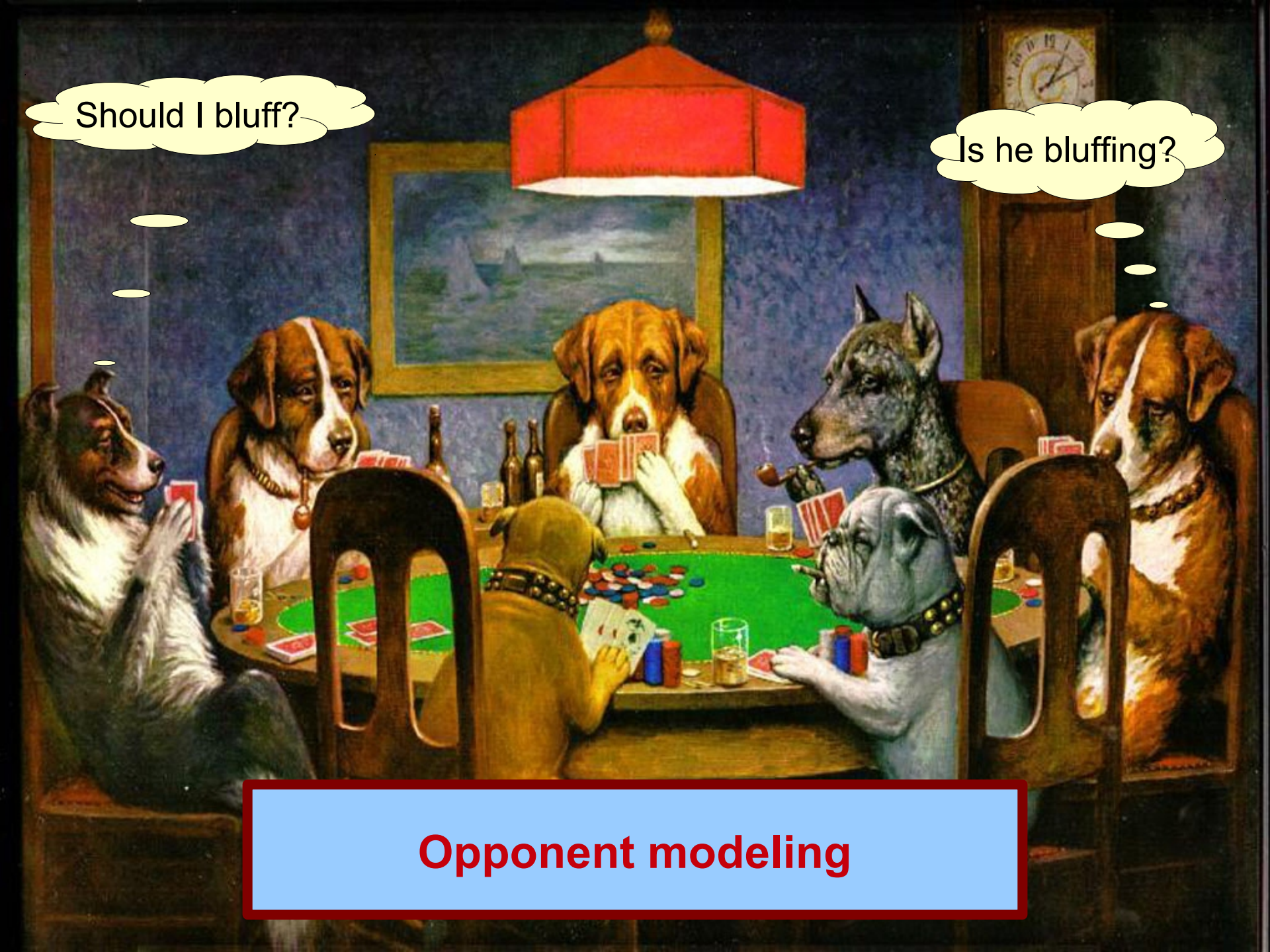


Guy Van den Broeck

KATHOLIEKE UNIVERSITEIT LEUVEN

DTAI
MACHINE LEARNING

Deceptive play


Should I bluff?
Is he bluffing?
Opponent modeling


Should I bluff?
Who has the Ace?
Is he bluffing?
Incomplete information


Should I bluff?
Who has the Ace?
What are the odds?
Is he bluffing?
Game of chance

Exploitation

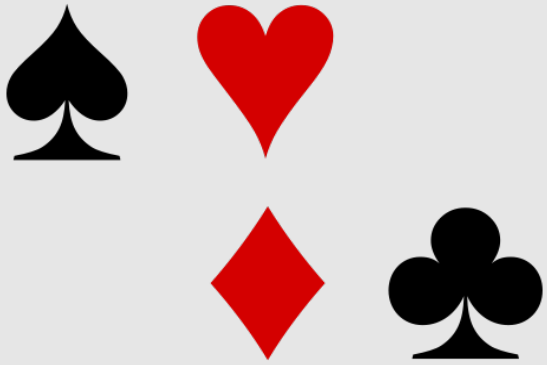
Huge state space

Risk management &
Continuous action space

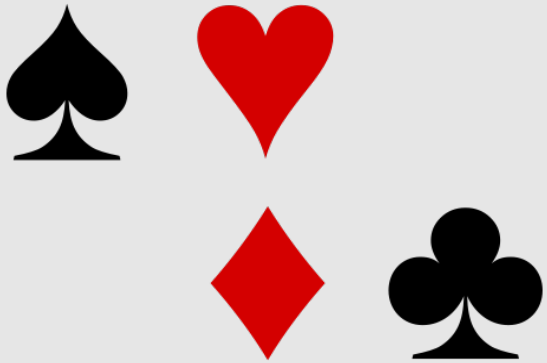
Take-Away Message:

We can solve all these problems!

Problem Statement



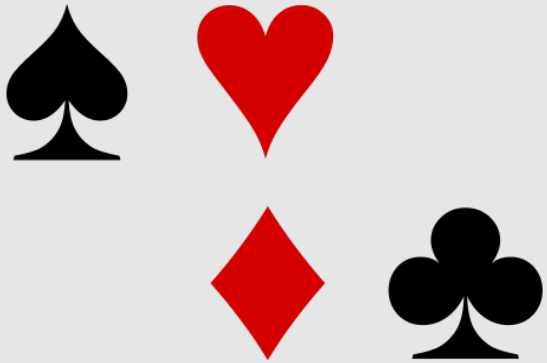
- A bot for Texas hold'em poker
  - No-Limit & > 2 players
    - Not done before!
  - Exploitative, not game theoretic
    - Game tree search + Opponent modeling
- Applies to any problem with either
  - incomplete information
  - non-determinism
  - continuous actions



Outline

- Overview approach
  - The Poker game tree
  - Opponent model
  - Monte-Carlo tree search
- Research challenges
  - Search
  - Opponent model
- Conclusion

Outline



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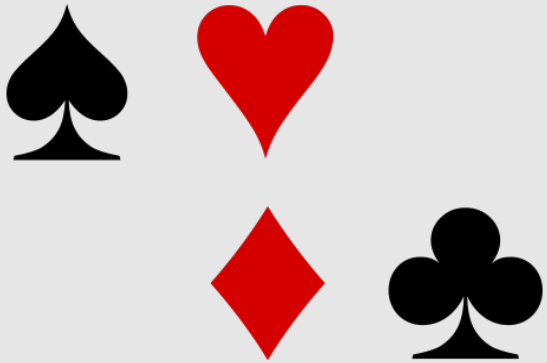

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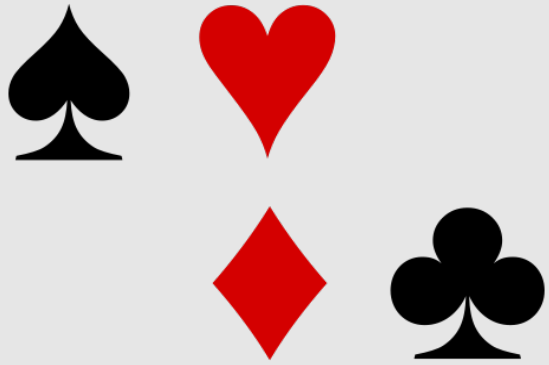
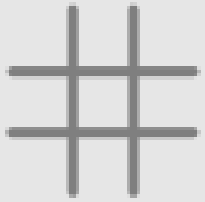
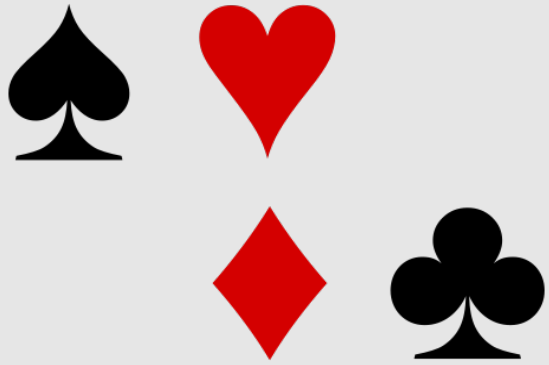
Poker Game Tree



- Minimax trees: deterministic
  - Tic-tac-toe, checkers, chess, go,…



Poker Game Tree



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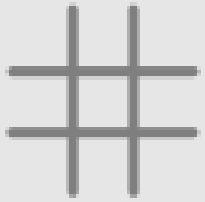
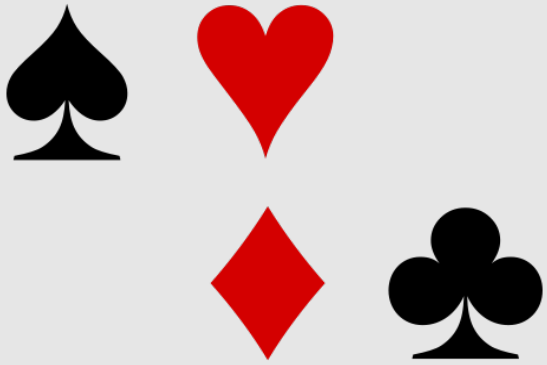


max *min*

- Expecti(mini)max trees: chance
  - Backgammon, …

max *min* *mix*

Poker Game Tree



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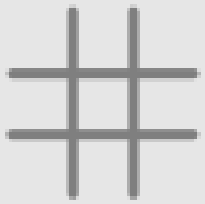


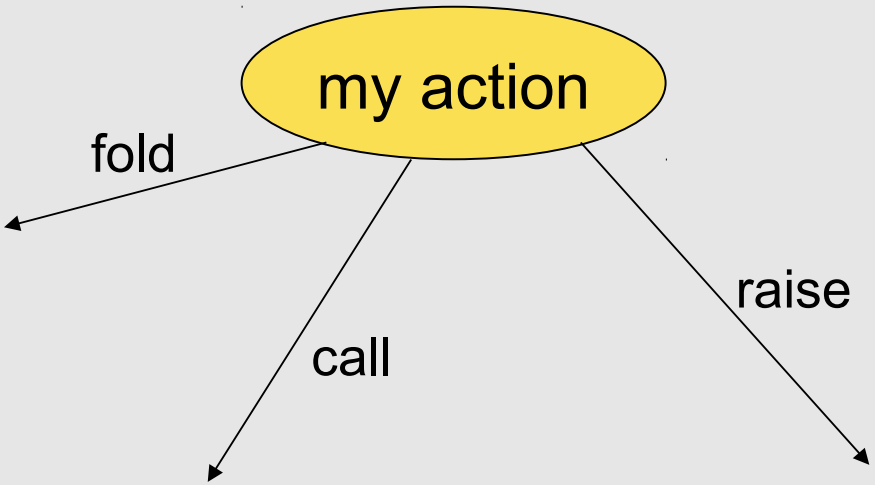
max *min*

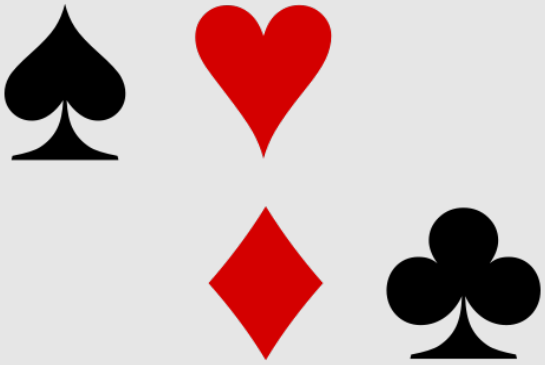
- Expecti(mini)max trees: chance
  - Backgammon, …

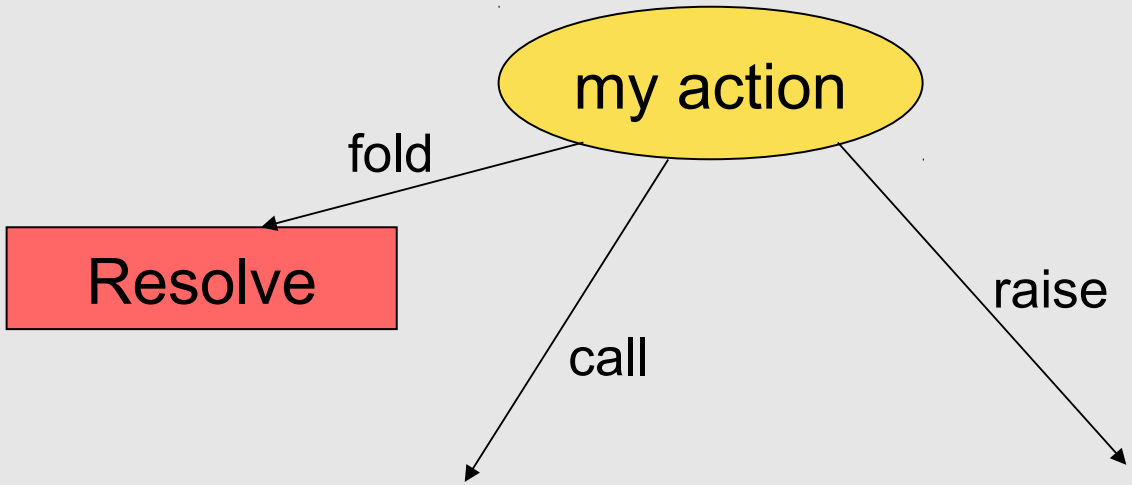
max *min* *mix*

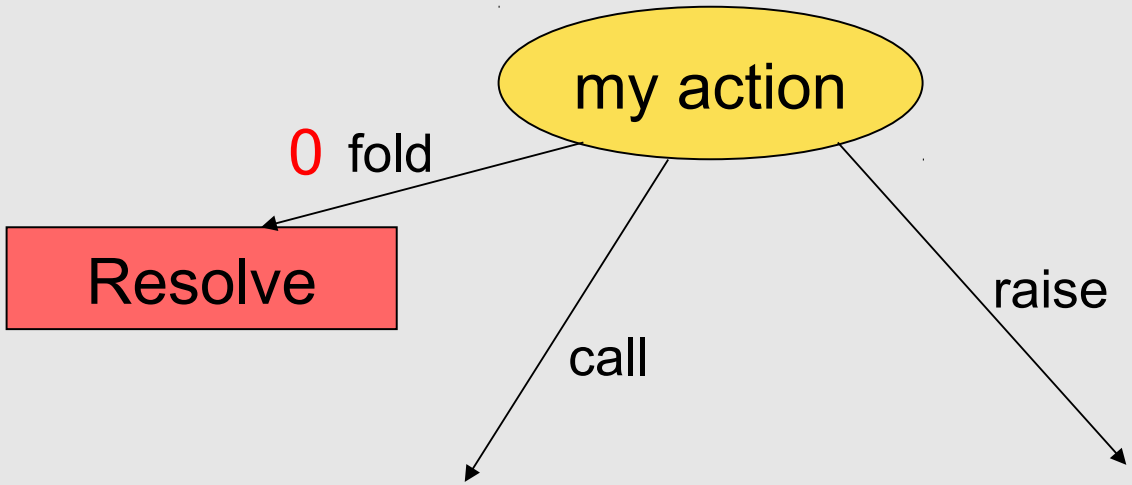
- Miximax trees: hidden information

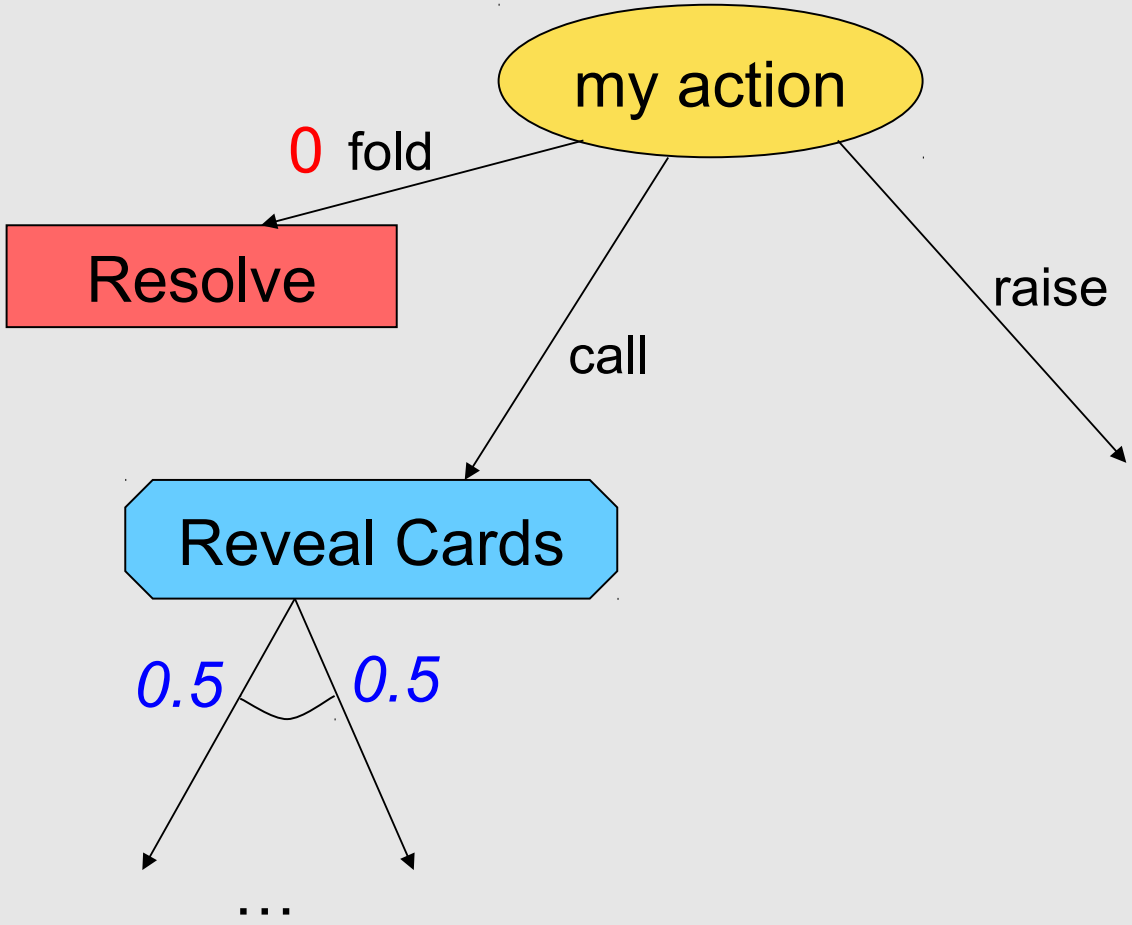
max *mix* *mix* + opponent model

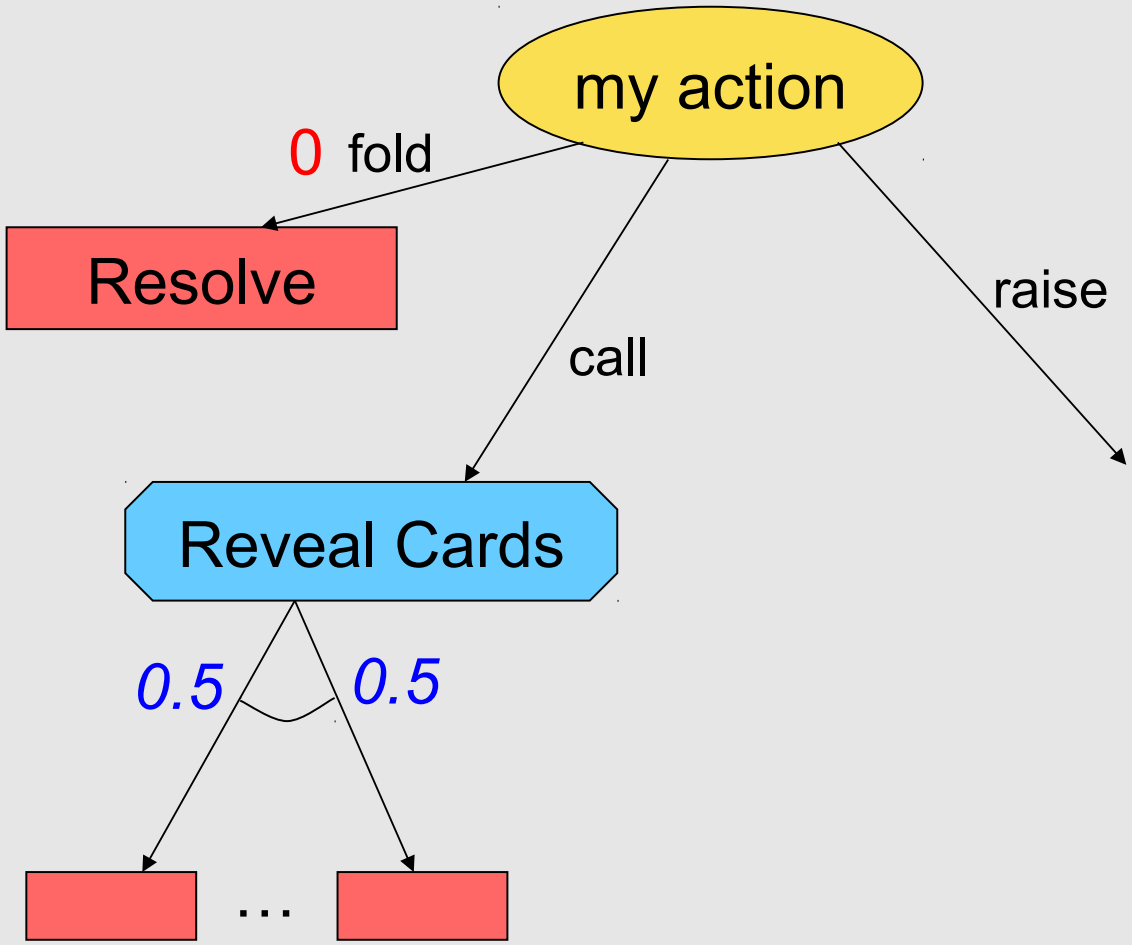

my action
fold
call
raise

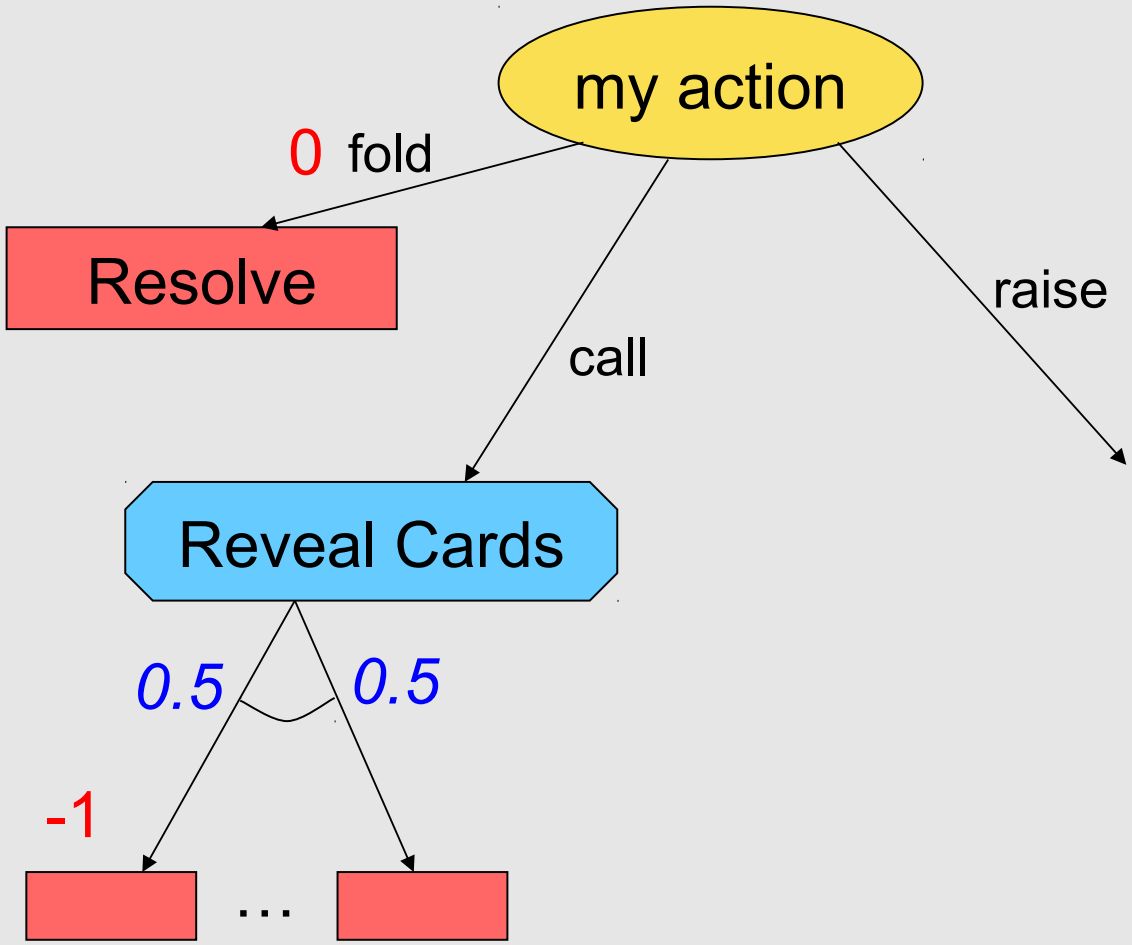


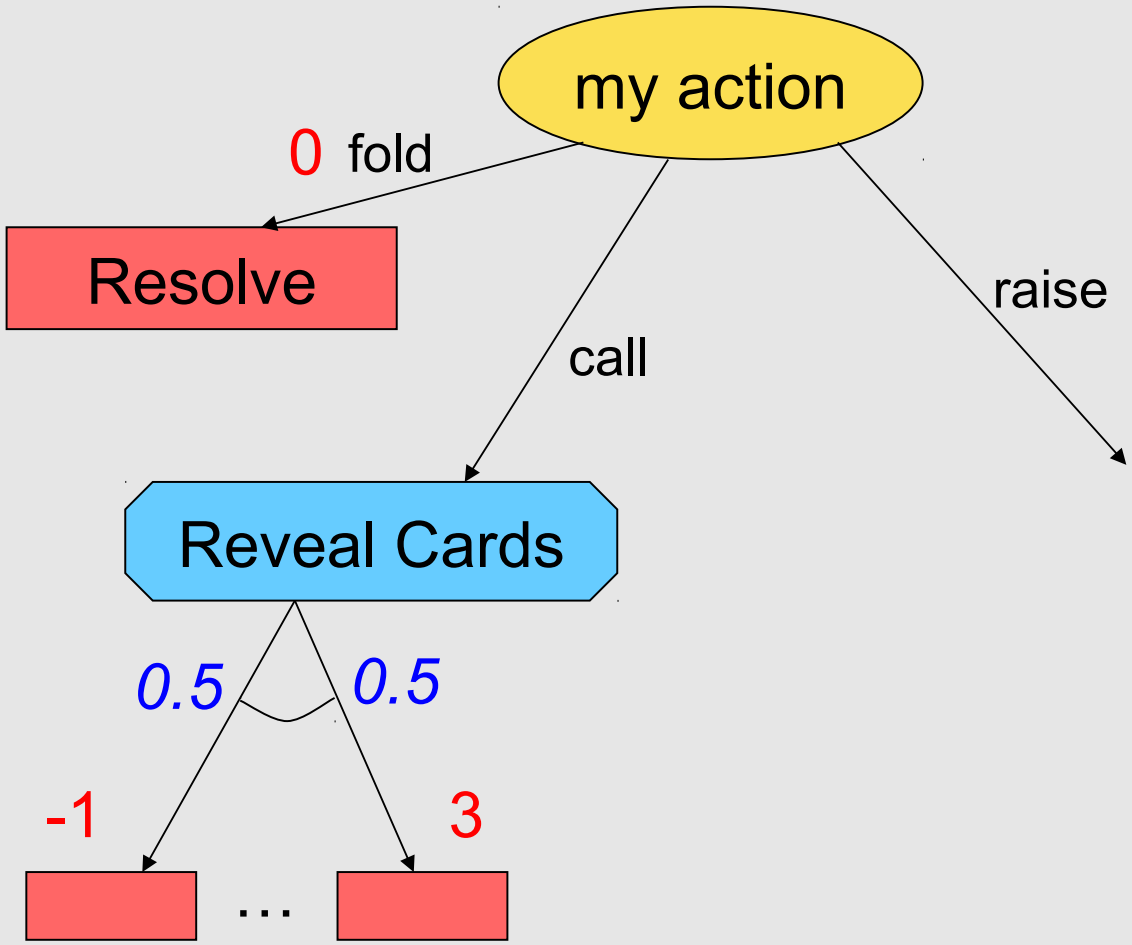

my action
fold
Resolve
call
raise

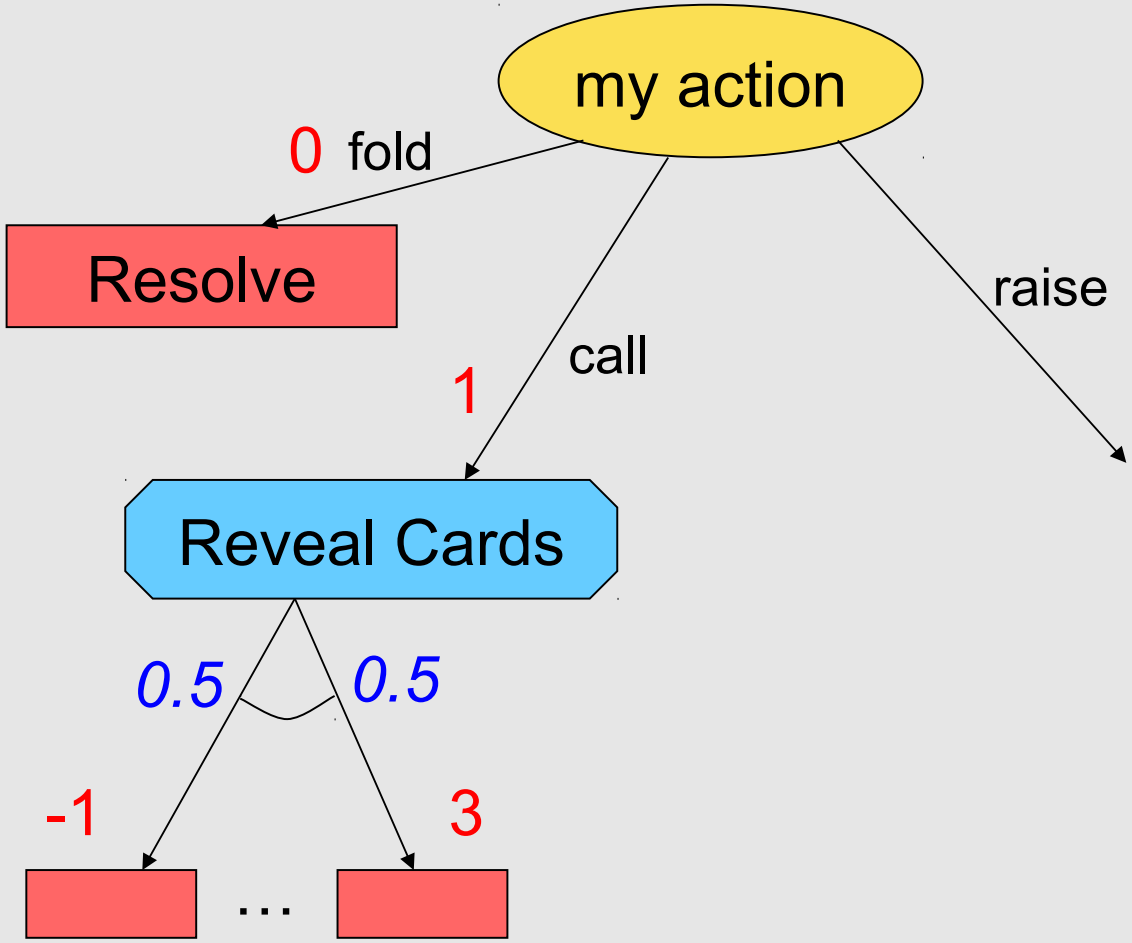

my action
0 fold
Resolve
call
raise

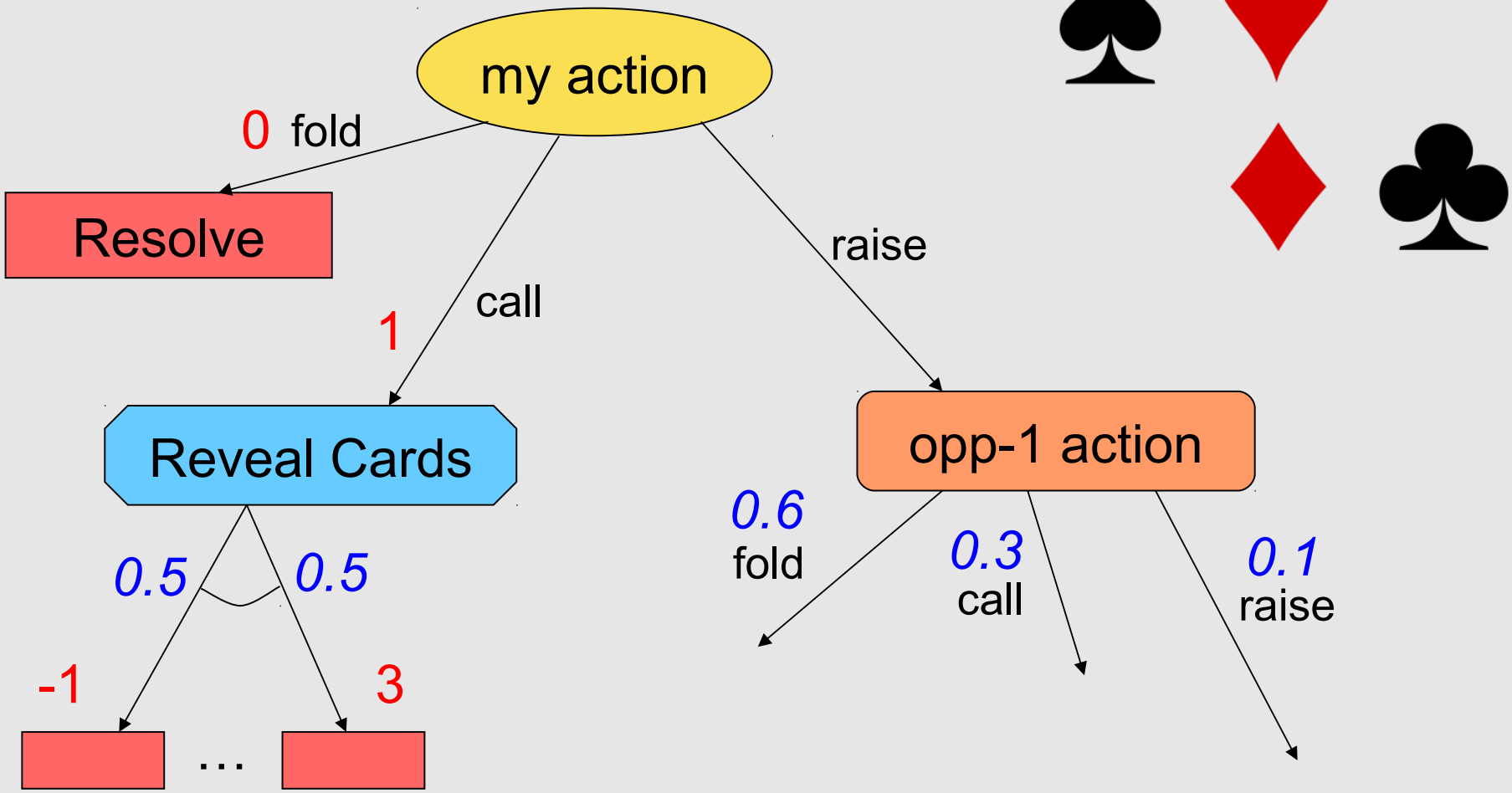

my action
0 fold
Resolve
call
raise
Reveal Cards
0.5
0.5
...

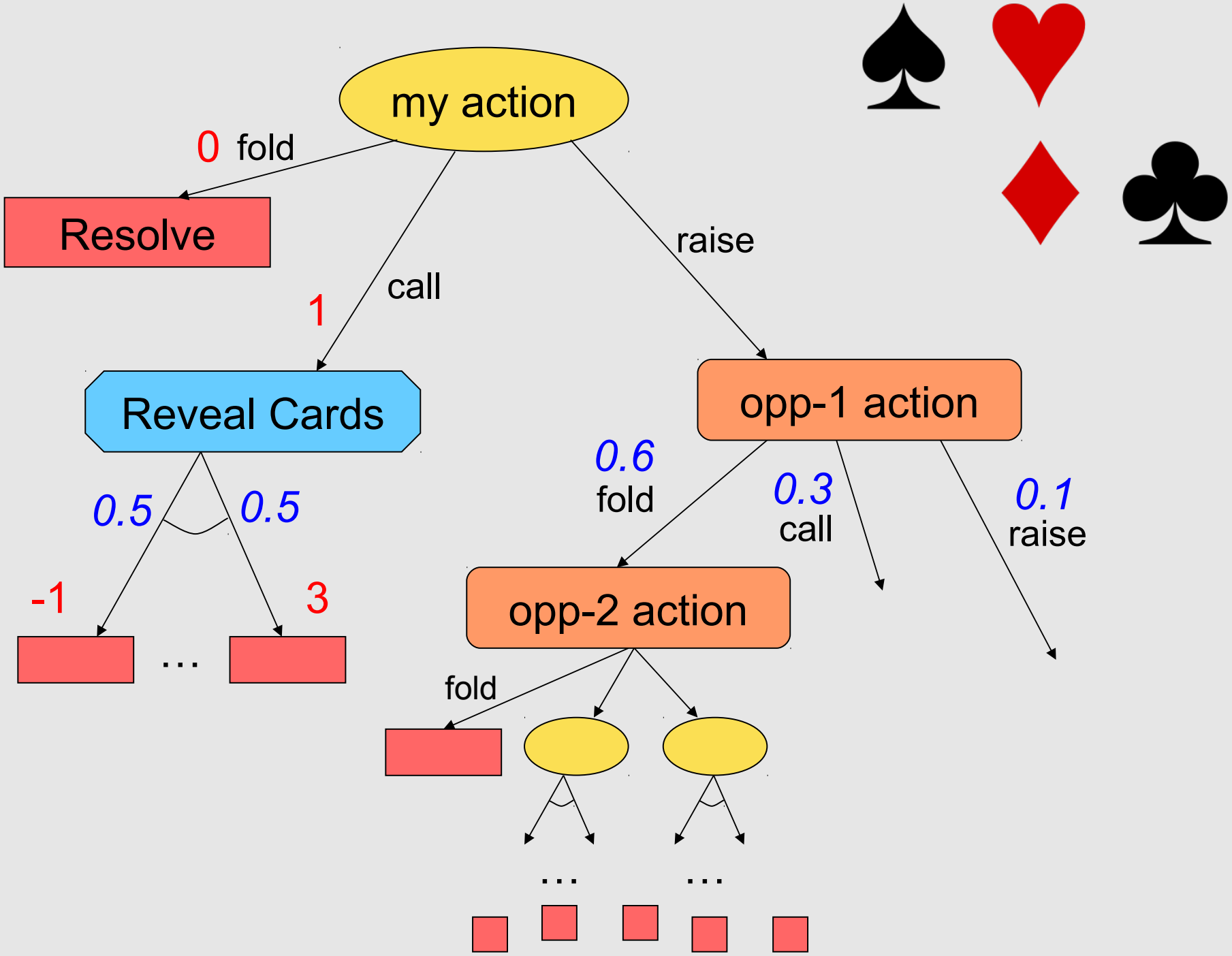

my action
0 fold
Resolve
call
raise
Reveal Cards
0.5
0.5
...

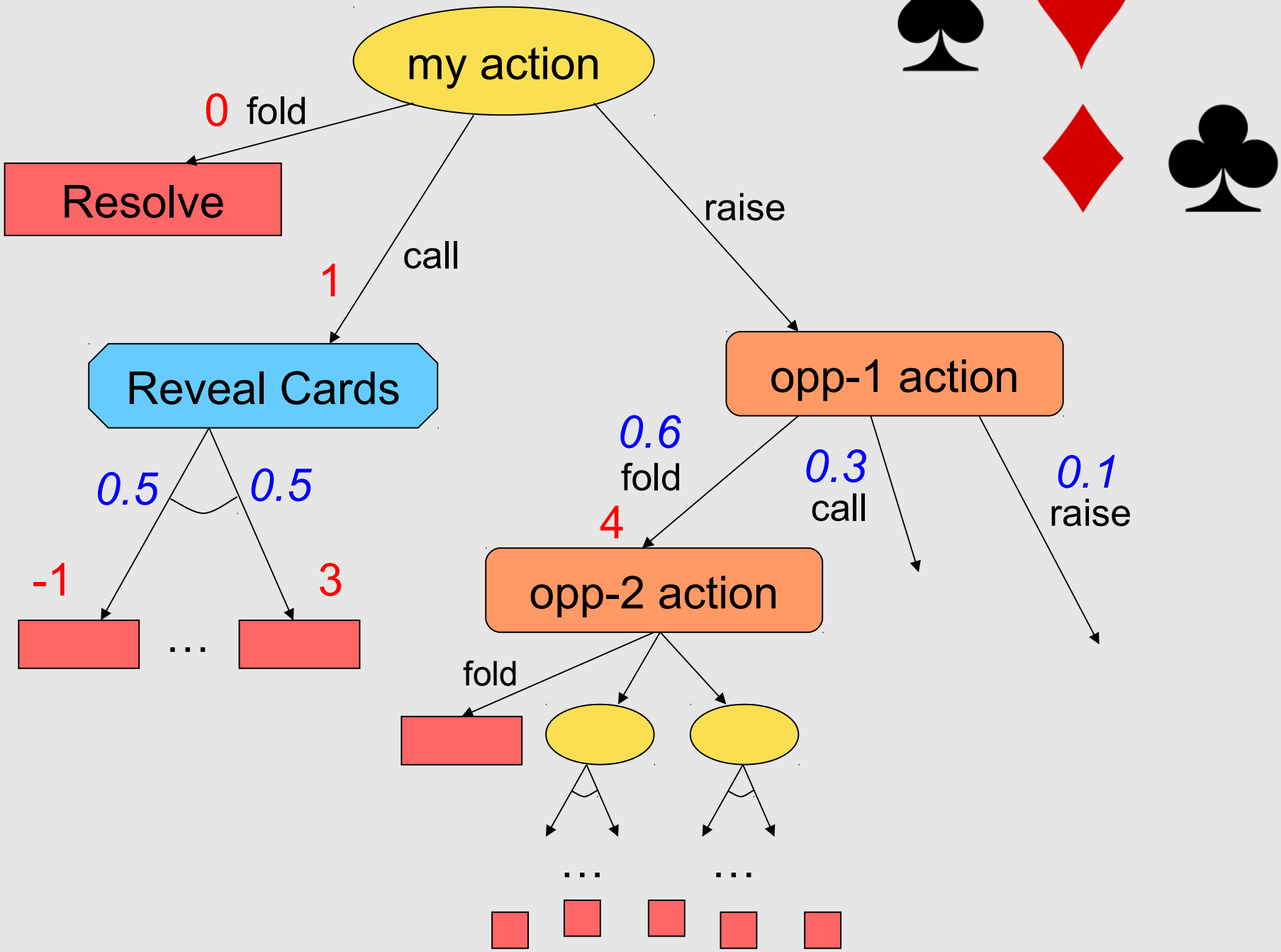

my action
0 fold
Resolve
call
raise
Reveal Cards
0.5
0.5
-1
...

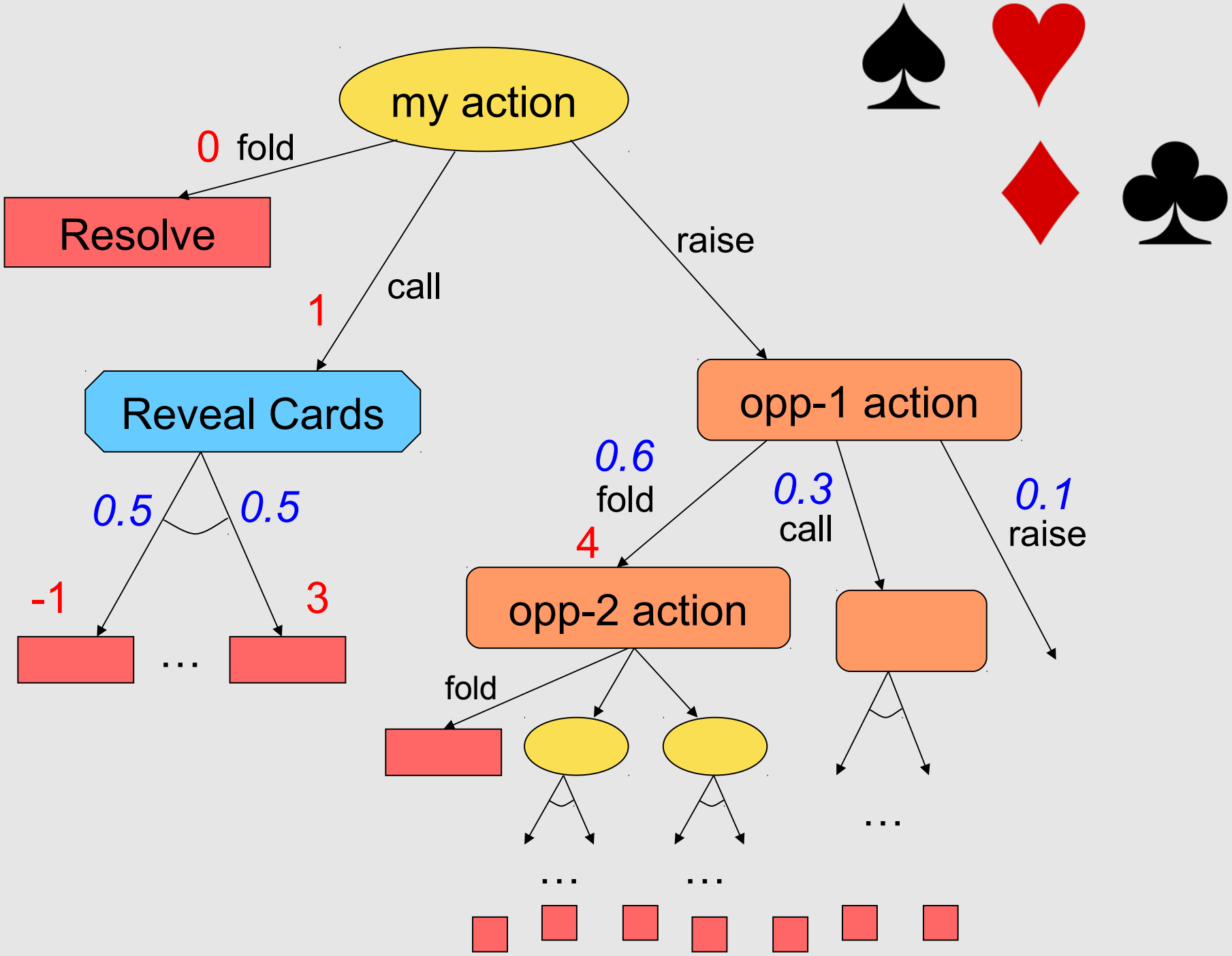

my action
0 fold
Resolve
call
raise
Reveal Cards
0.5
0.5
-1
3
...


my action
0 fold
Resolve
raise
1
call
Reveal Cards
0.5
0.5
-1
3
...


my action
0 fold
Resolve
call
1
raise
Reveal Cards
opp-1 action
0.5
0.5
-1
3
...
0.6
fold
0.3
call
0.1
raise


my action
0 fold
Resolve
1
call
raise
Reveal Cards
opp-1 action
0.5
0.5
-1
3
...
0.6
fold
0.3
call
0.1
raise
opp-2 action
fold
...
...


my action
0 fold
Resolve
1
call
raise
Reveal Cards
opp-1 action
0.5
0.5
-1
3
...
0.6
fold
4
0.3
call
0.1
raise
opp-2 action
fold
...
...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call
raise
Reveal Cards
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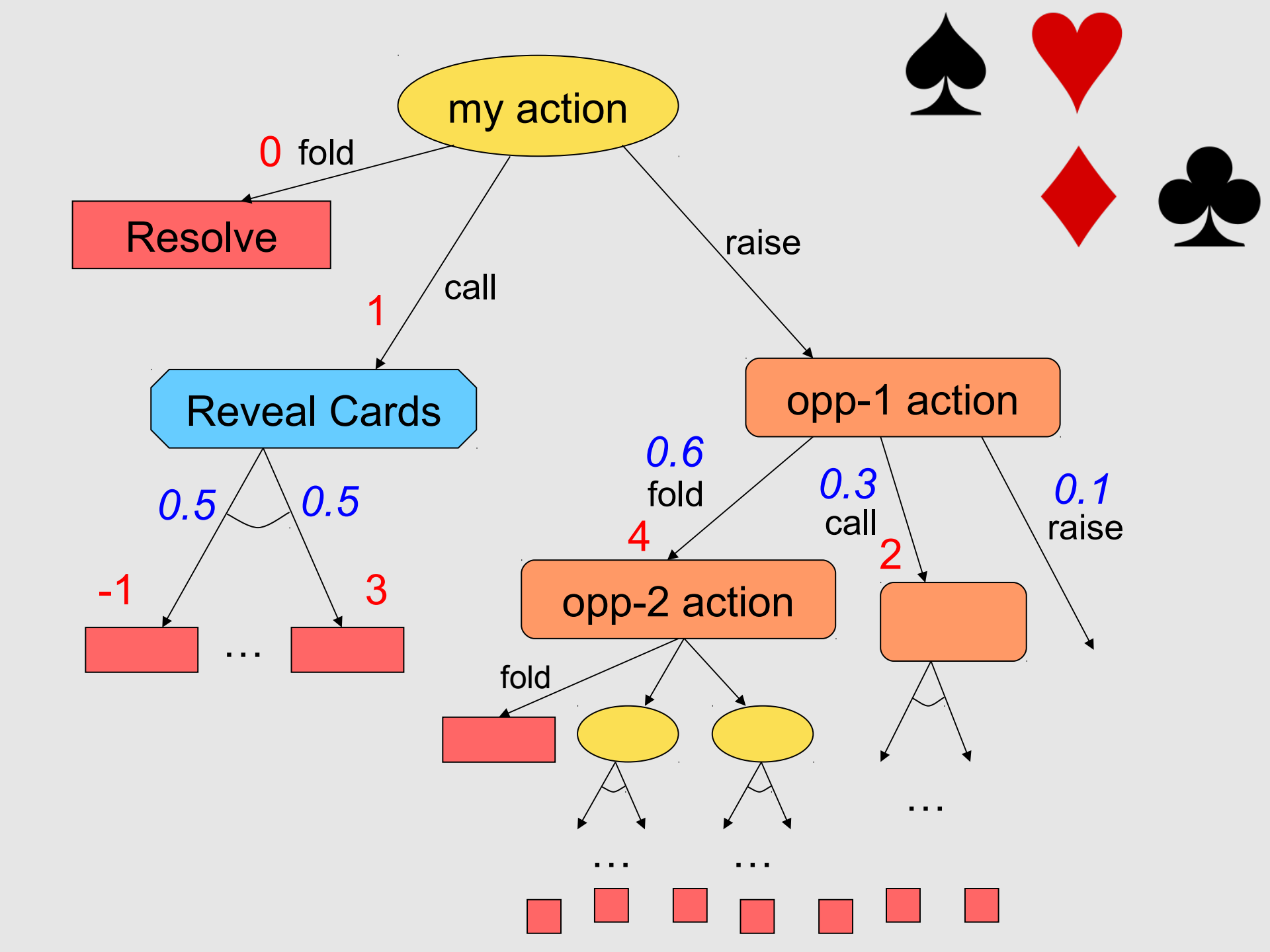
0.5
0.5
-1
3
0.6
fold
4
0.3
call
0.1
raise
opp-2 action
fold


my action
0 fold
Resolve
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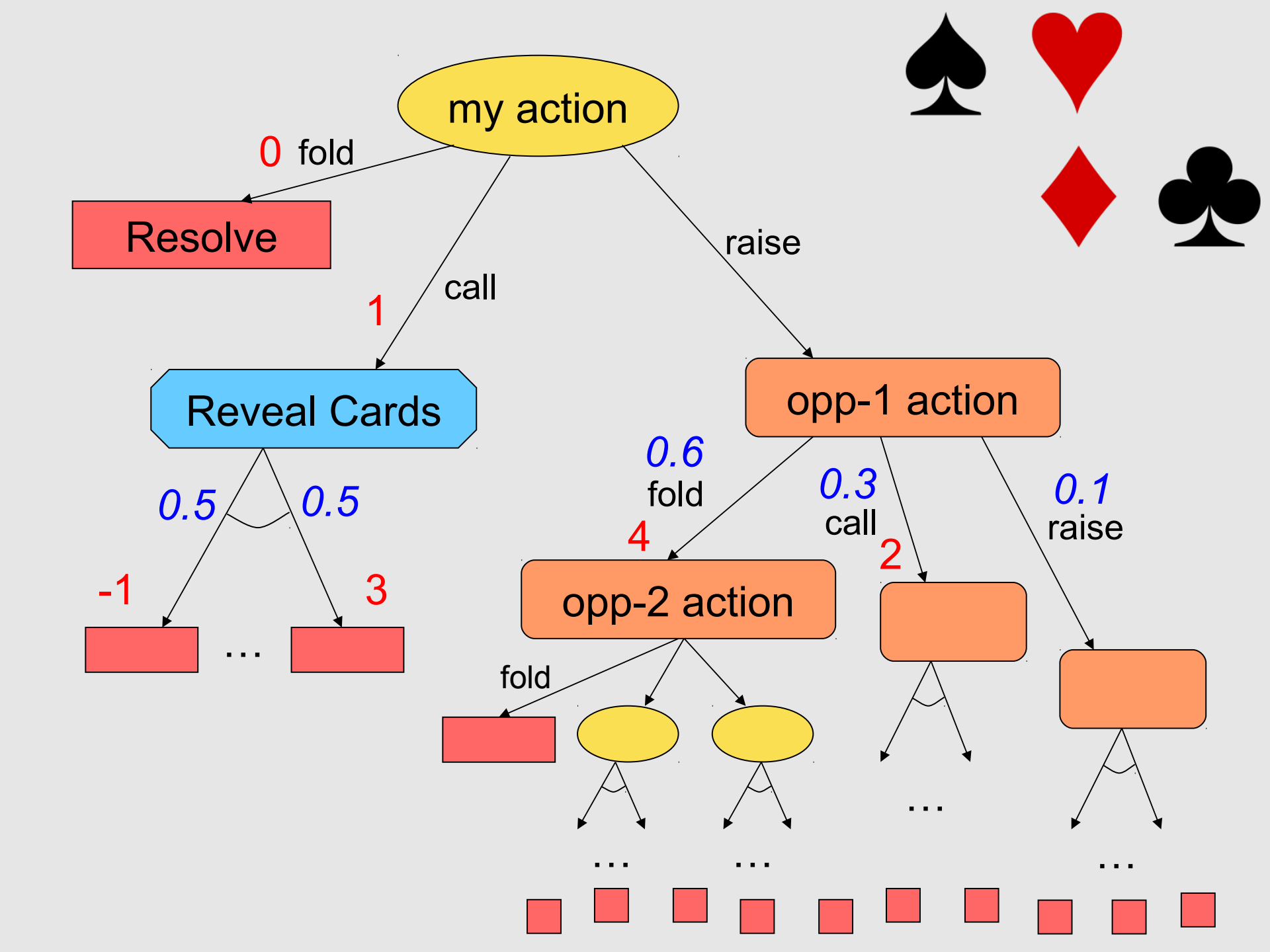
call
raise
Reveal Cards
opp-1 action
0.5
0.5
-1
3
0.6
fold
4
0.3
call
2
0.1
raise
opp-2 action
fold


my action
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Resolve
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call
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Reveal Cards
opp-1 action
0.5
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fold
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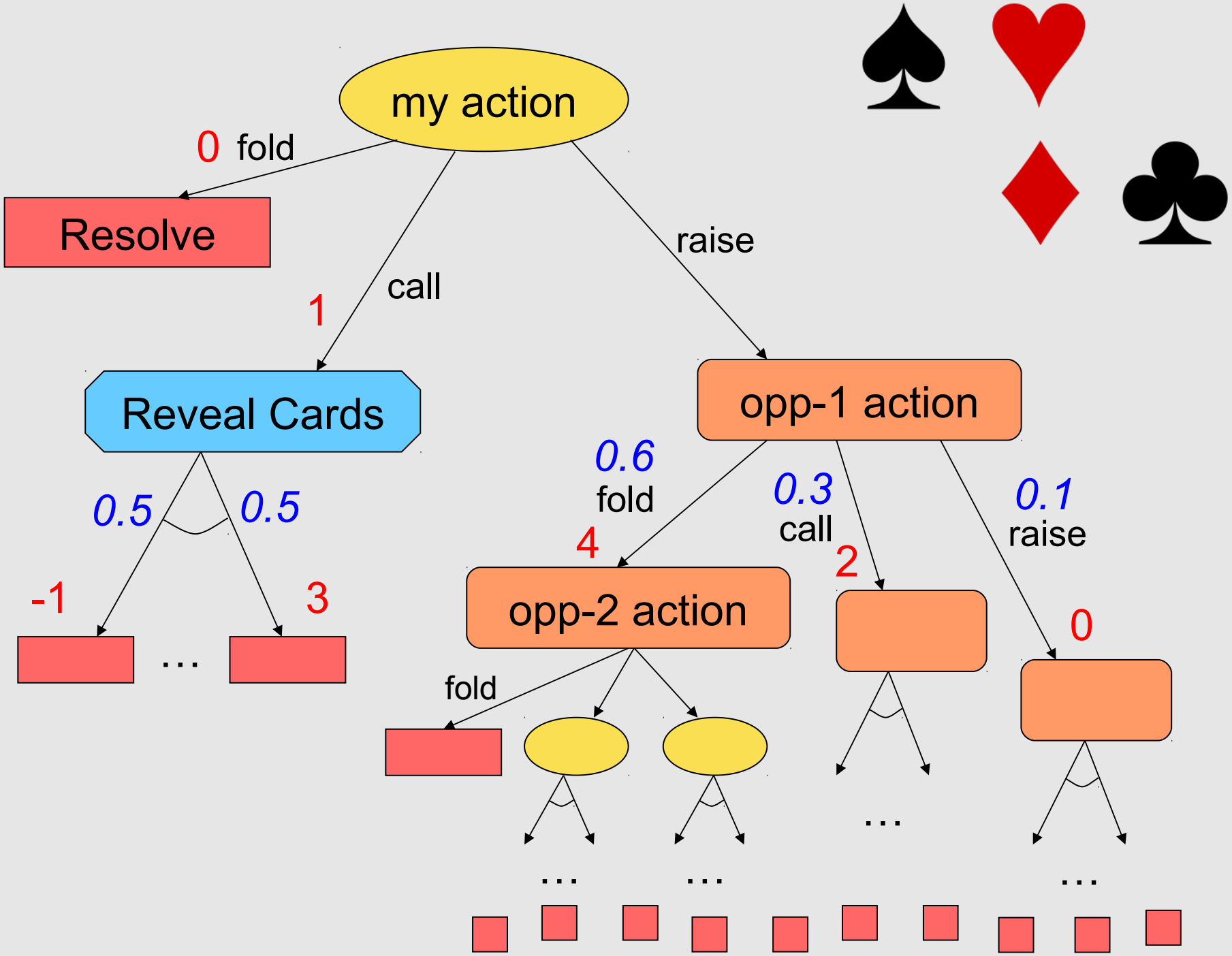
2
0.1
raise
opp-2 action
fold


my action
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0.6
fold
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raise
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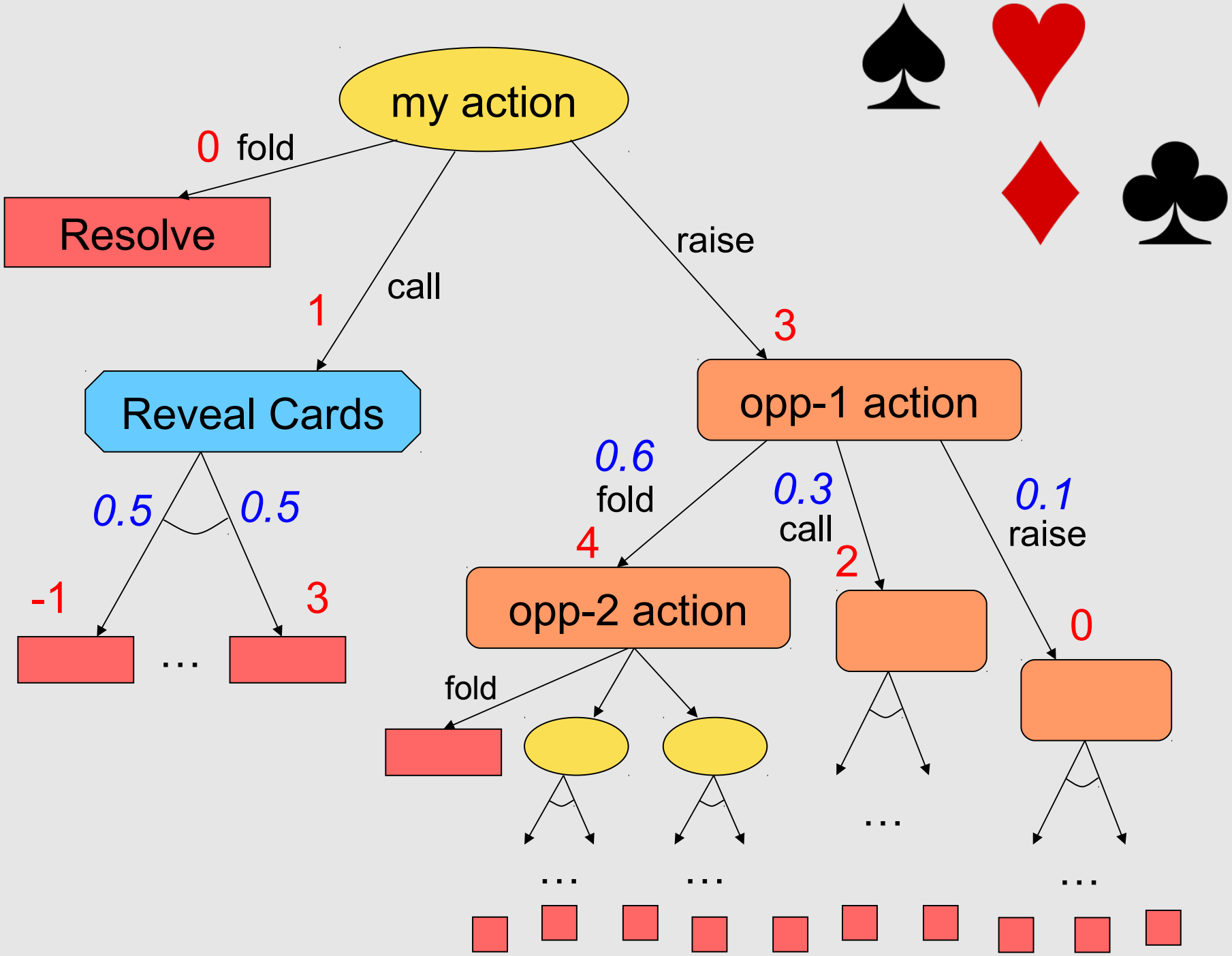
opp-2 action
fold


my action
0 fold
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1
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opp-2 action
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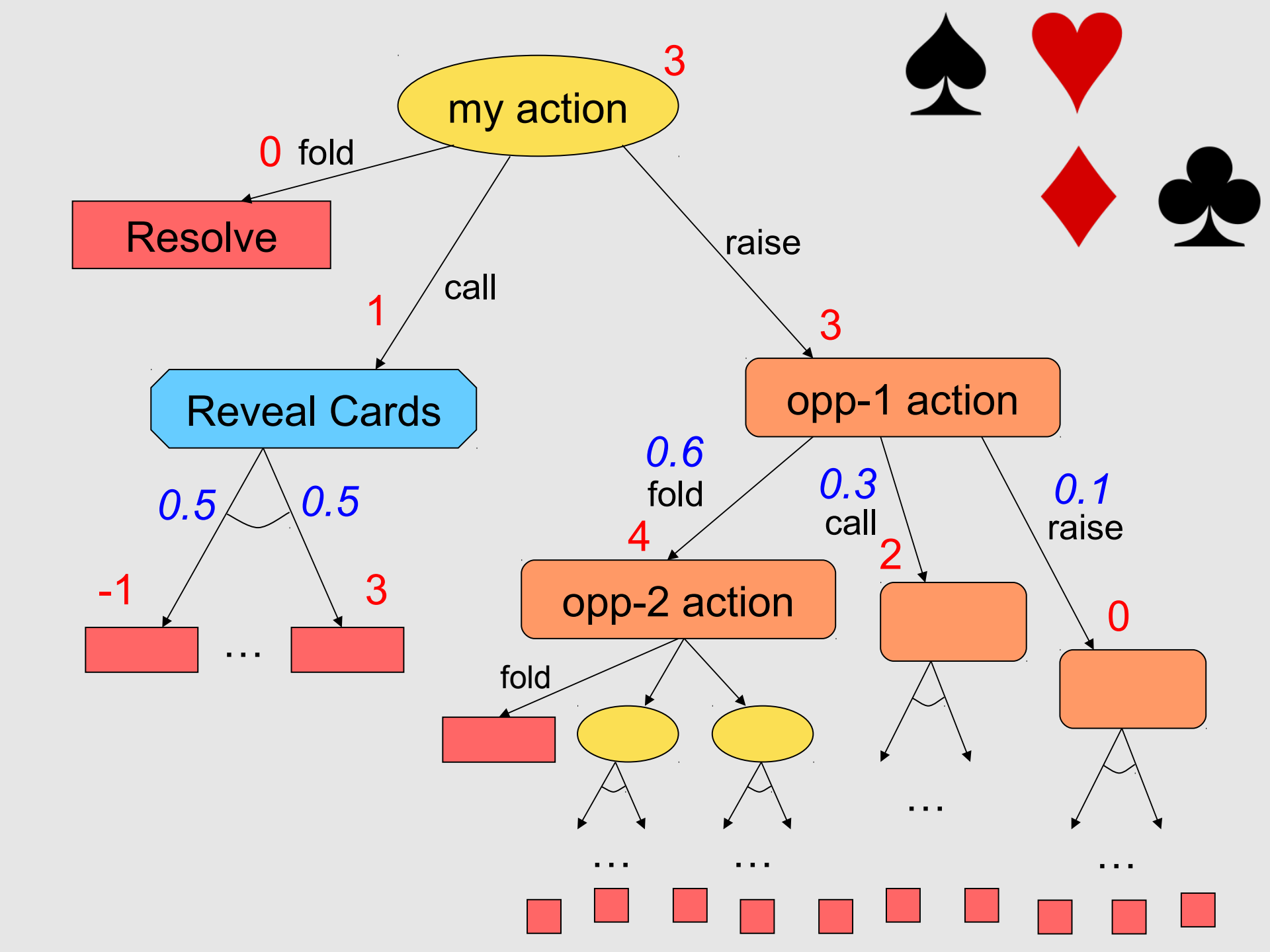
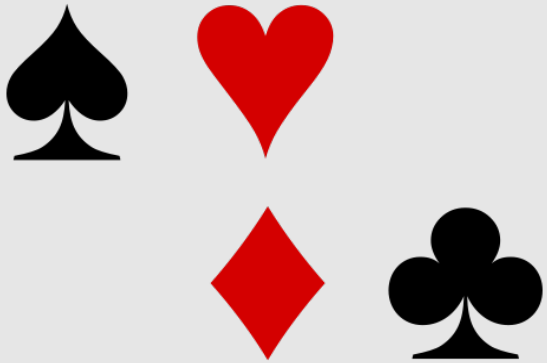
call
2
0.1
raise
0


my action
3
0 fold
Resolve
call
1
raise
3
Reveal Cards
opp-1 action
0.5
0.5
-1
3
0.6
fold
4
0.3
call
2
0.1
raise
0
opp-2 action
fold



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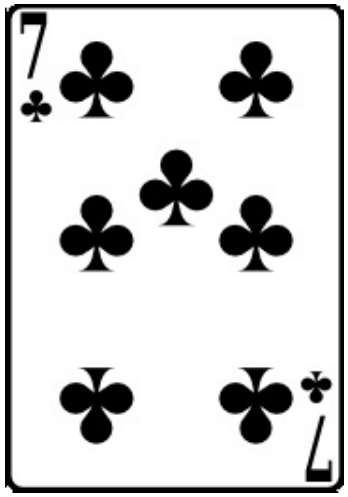
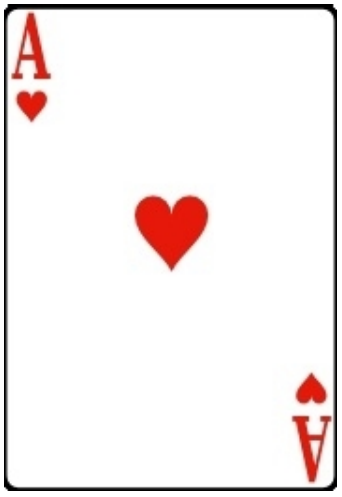
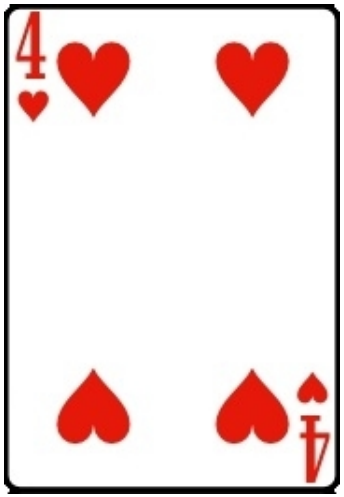
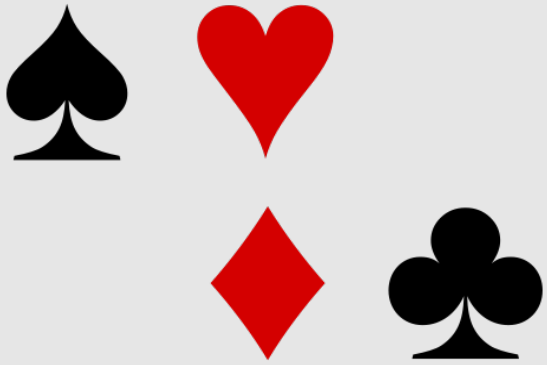
Short Experiment


Opponent Model



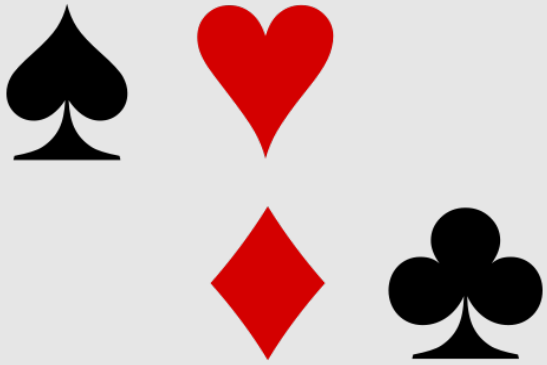
- Set of probability trees
- Weka's M5'
- Separate model for
  - Actions

    $$P(A_i|A_0 \ldots A_{i-1}, C_0 \ldots C_i)$$

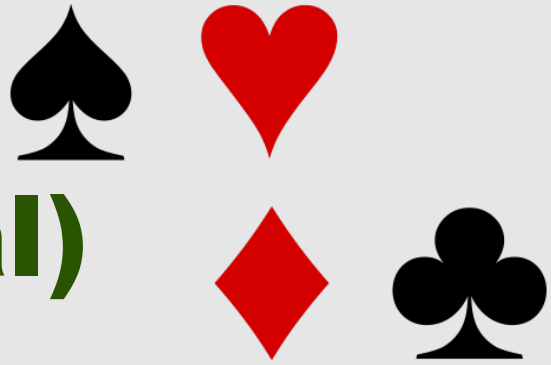
  - Hand cards at showdown

    $$P(H|A_0 \ldots A_n, C_0 \ldots C_n)$$

Fold Probability

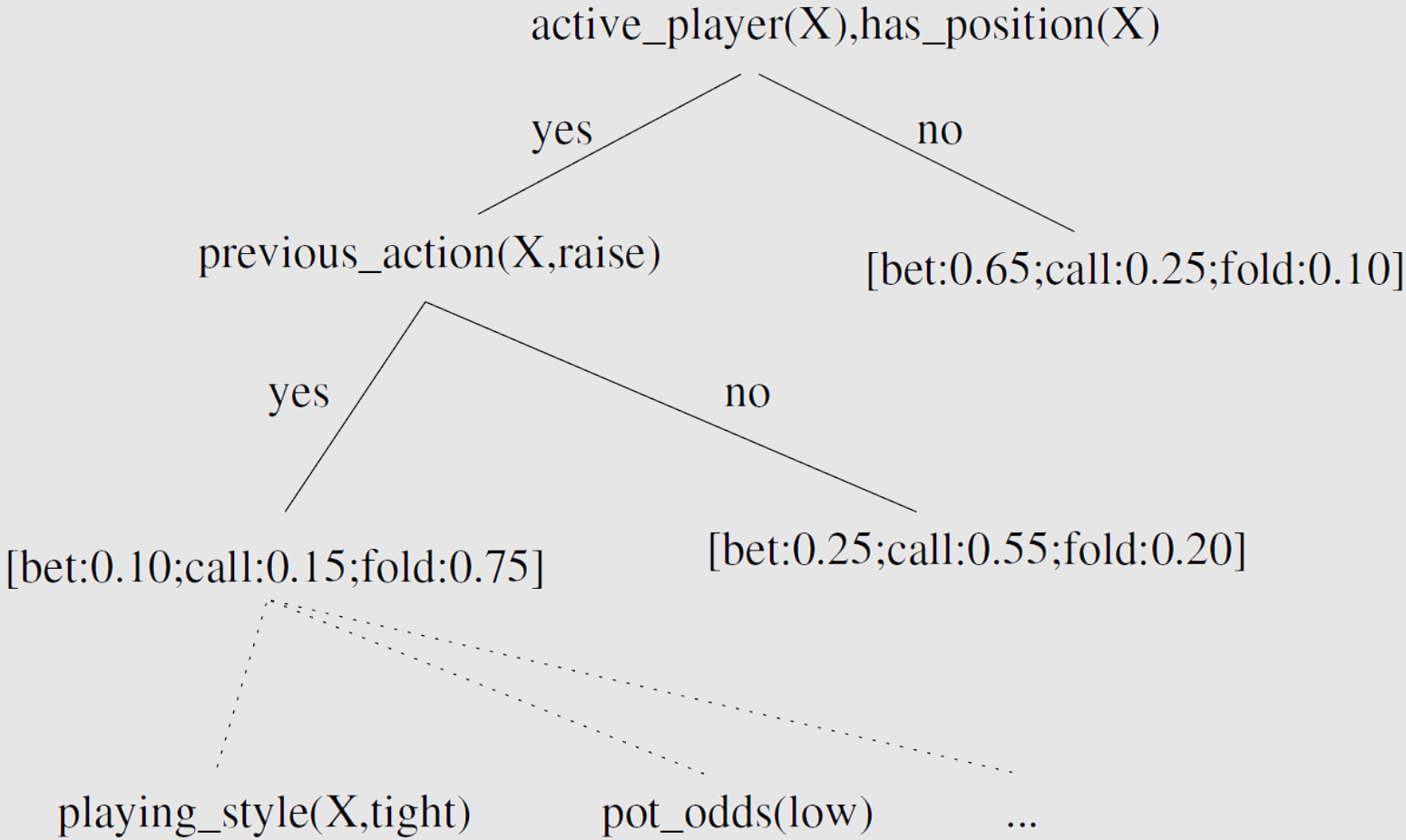


```
nbAllPlayerRaises <= 1.5 :
|    callFrequency <= 0.128 :
|    |    nbActionsThisRound <= 2.5 :
|    |    |    potOdds <= 0.28 :
|    |    |    |    AF <= 2.585 : 0.6904
|    |    |    |    AF >  2.585 :
|    |    |    |    |    potSize <= 3.388 :
|    |    |    |    |    |    round=flop <= 0.5 : 0.8068
|    |    |    |    |    |    round=flop >  0.5 : 0.6896
|    |    |    |    |    potSize >  3.388 : 0.8198
|    |    |    potOdds >  0.28 :
|    |    |    |    stackSize <= 97.238 :
|    |    |    |    |    callFrequency <= 0.038 : 0.8838
|    |    |    |    |    callFrequency >  0.038 :
|    |    |    |    |    |    round=flop <= 0.5 : 0.8316
|    |    |    |    |    |    round=flop >  0.5 :
|    |    |    |    |    |    |    nbSeatedPlayers <= 7.5 : 0.6614
|    |    |    |    |    |    |    nbSeatedPlayers >  7.5 : 0.7793
|    |    |    |    stackSize >  97.238 :
|    |    |    |    |    potSize <= 4.125 :
|    |    |    |    |    |    foldFrequency <= 0.813 : 0.7839
|    |    |    |    |    |    foldFrequency >  0.813 : 0.9037
|    |    |    |    |    potSize >  4.125 : 0.8623
|    |    nbActionsThisRound >  2.5 :
|    |    |    potOdds <= 0.218 :
|    |    |    |    callFrequency <= 0.067 : 0.8753
|    |    |    |    callFrequency >  0.067 : 0.7661
|    |    |    potOdds >  0.218 :
|    |    |    |    AF <= 2.654 : 0.8818
|    |    |    |    AF >  2.654 : 0.921
```



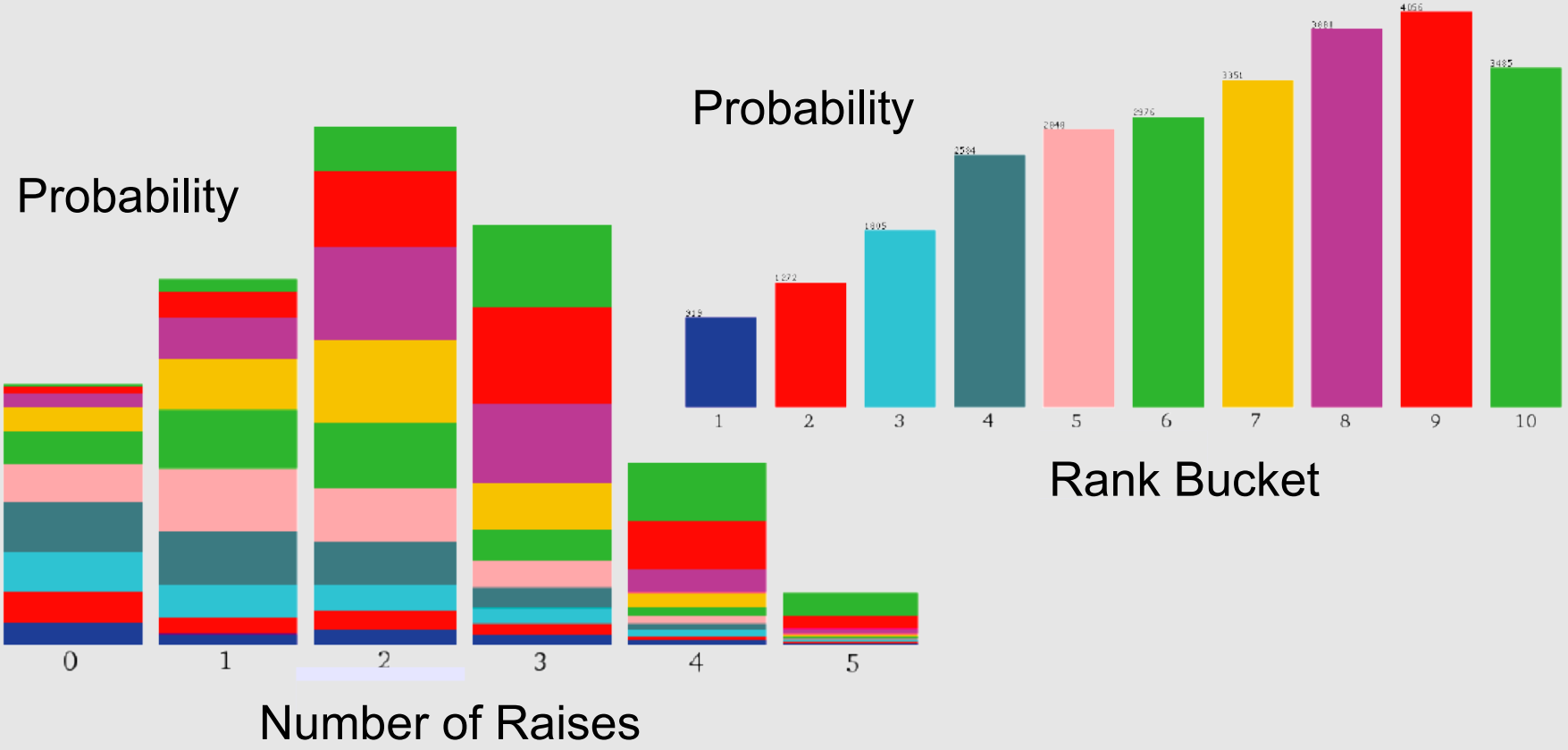
(Can also be relational)

- Tilde probability tree [Ponsen08]



Opponent Ranks

- Learn distribution of hand ranks at showdown



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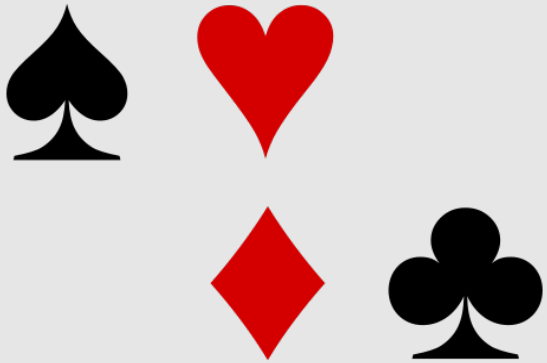

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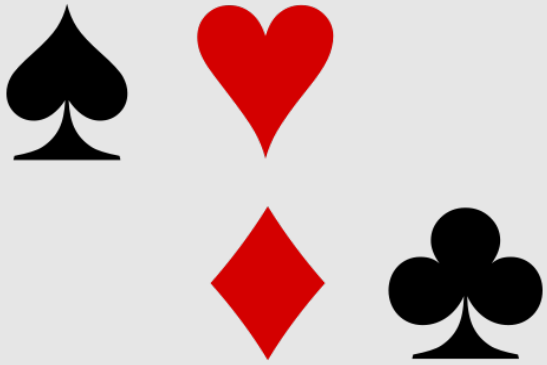
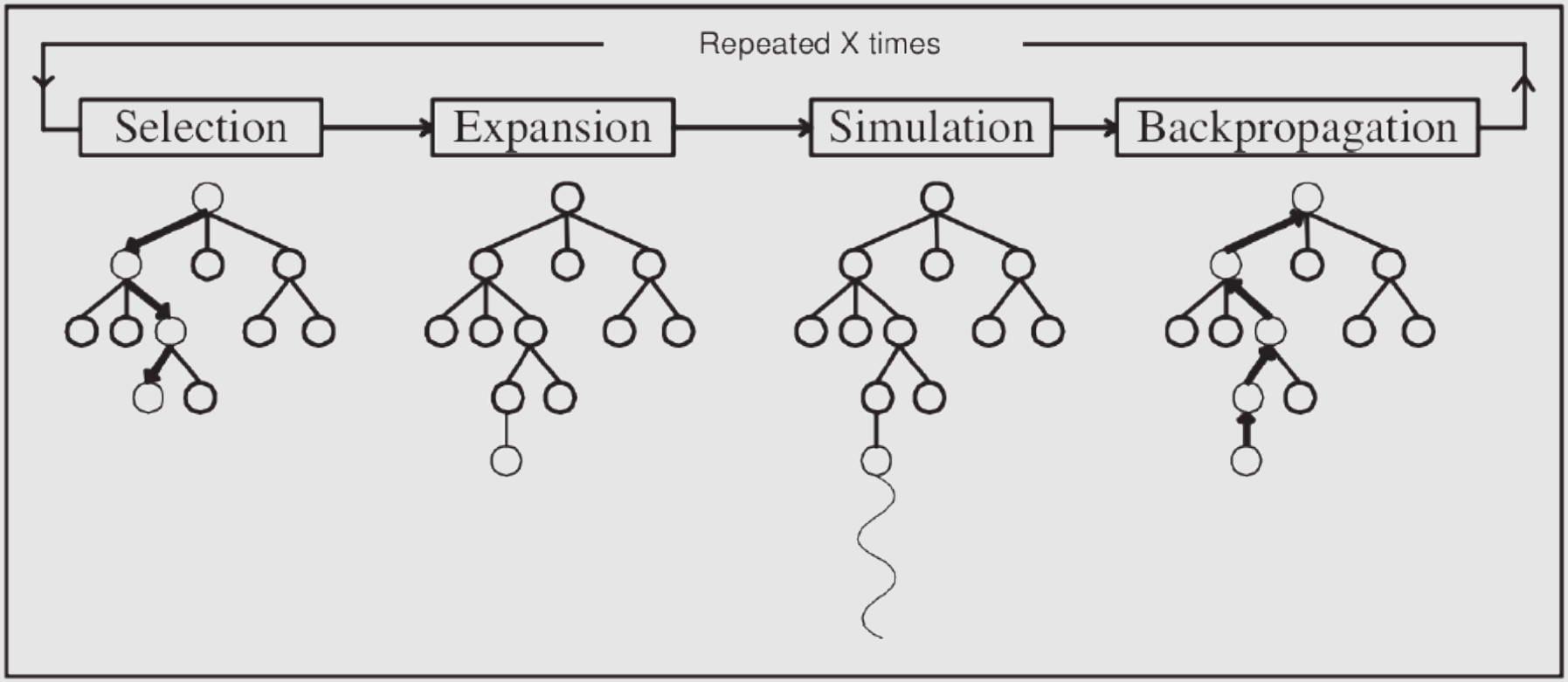
Traversing the tree



- Limit Texas Hold'em
  - 10^{18} nodes
  - Fully traversable
- No-limit
  - $>10^{71}$ nodes
  - Too large to traverse
  - Sampled, not searched
  - Monte-Carlo Tree Search

Monte-Carlo Tree Search

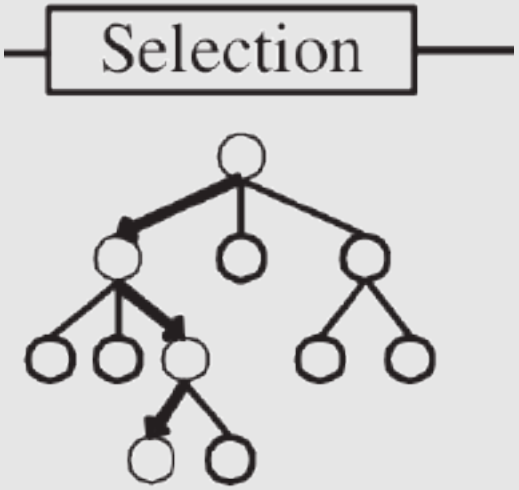


[Chaslot08]

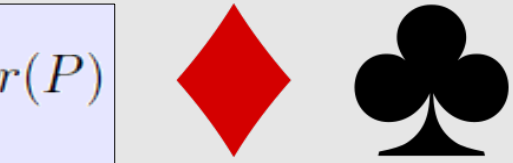
Selection

In each node:

$\hat{V}(P)$	is an estimate of the reward $r(P)$
$\mathrm{T}(P)$	is the number of samples



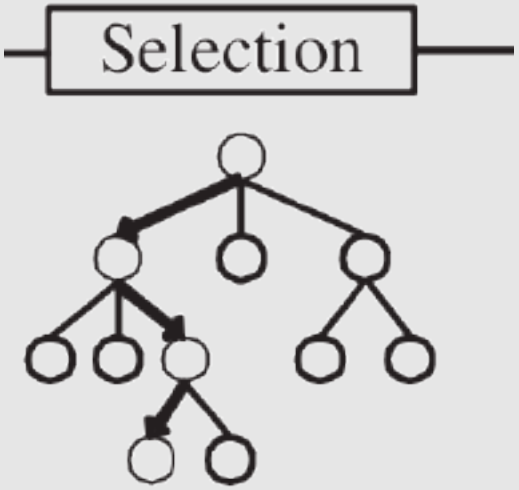
Selection



In each node:

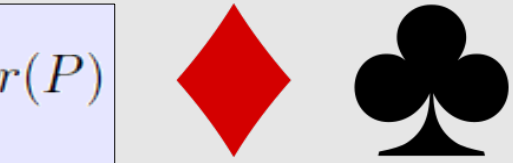
$\hat{V}(P)$	is an estimate of the reward $r(P)$
$\mathrm{T}(P)$	is the number of samples

- UCT (Multi-Armed Bandit)



$$\hat{V}(c_i) + C\sqrt{\frac{\ln \mathrm{T}(P)}{\mathrm{T}(c_i)}}$$

Selection



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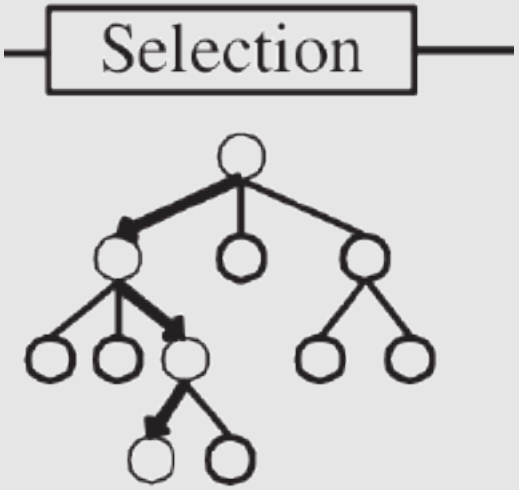
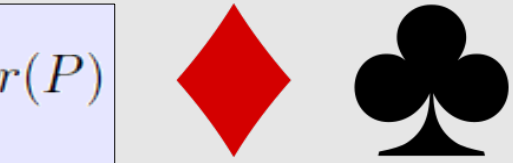


$$\hat{V}(c_i) + C\sqrt{\frac{\ln \mathrm{T}(P)}{\mathrm{T}(c_i)}}$$

exploitation

Selection



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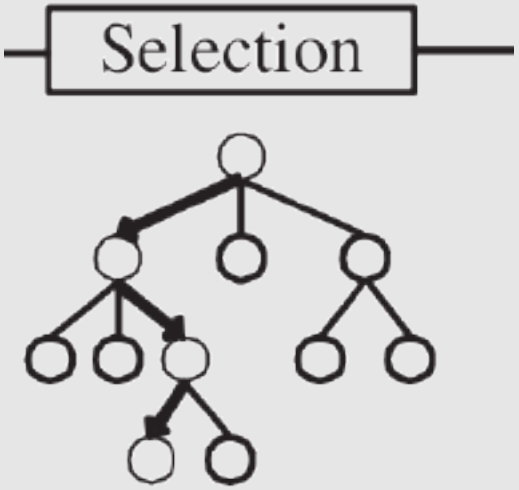

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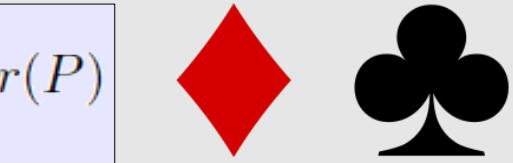
exploitation

exploration

Selection



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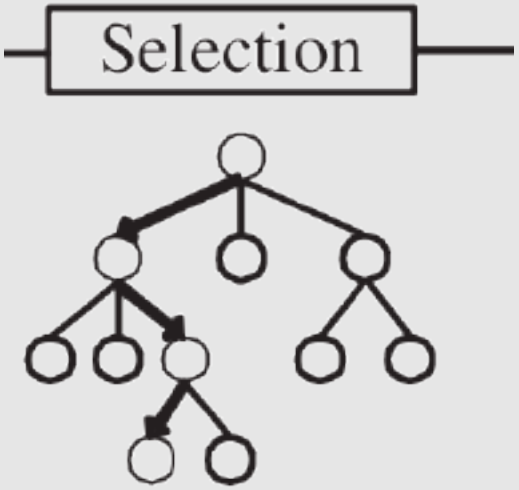


$$\hat{V}(c_i) + C\sqrt{\frac{\ln \mathrm{T}(P)}{\mathrm{T}(c_i)}}$$

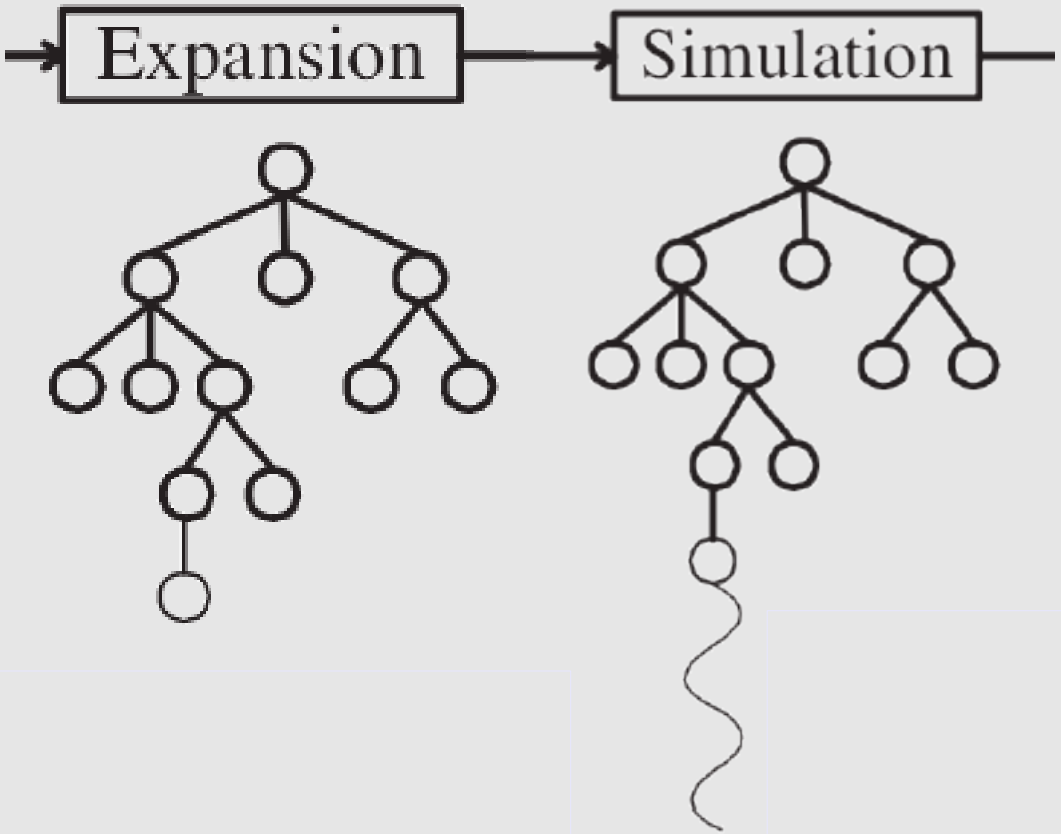
exploitation

exploration

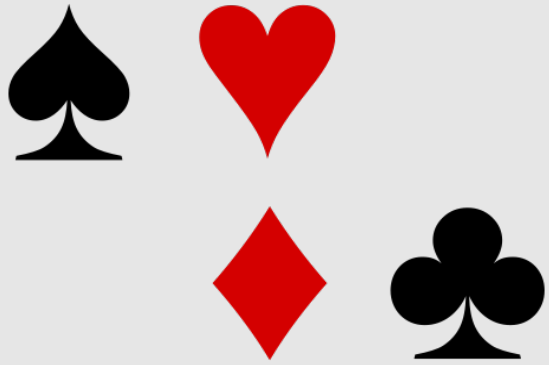
- CrazyStone

$$\mathrm{P}(c_i) \sim \exp\left(-2.4\frac{\hat{V}(c_{best}) - \hat{V}(c_i)}{\sqrt{2(\overline{\sigma}(c_{best})^2 + \overline{\sigma}(c_i)^2)}}\right)$$

Expansion
Simulation

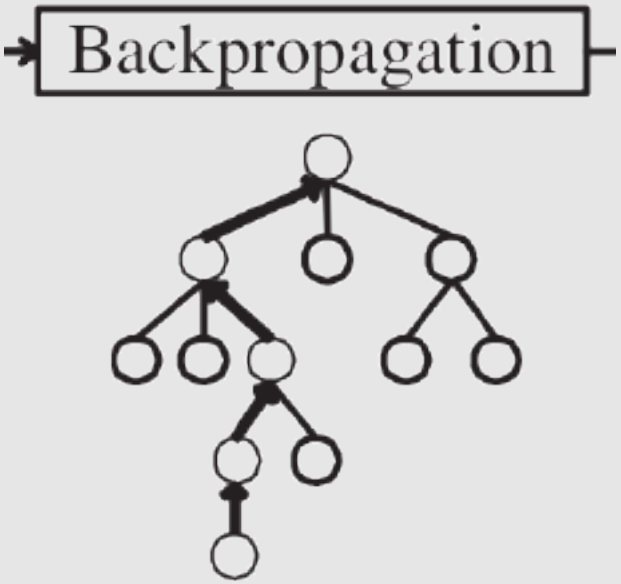


Backpropagation

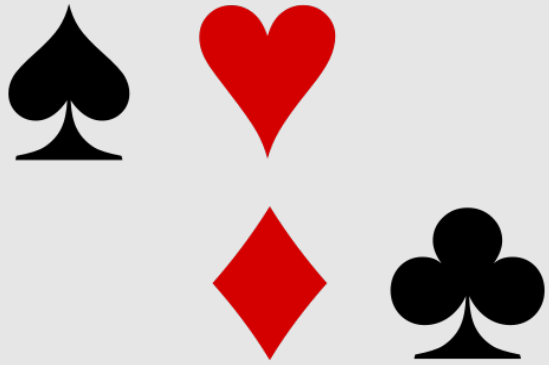


$\hat{V}(P)$ is an estimate of the reward $r(P)$

$\mathrm{T}(P)$ is the number of samples



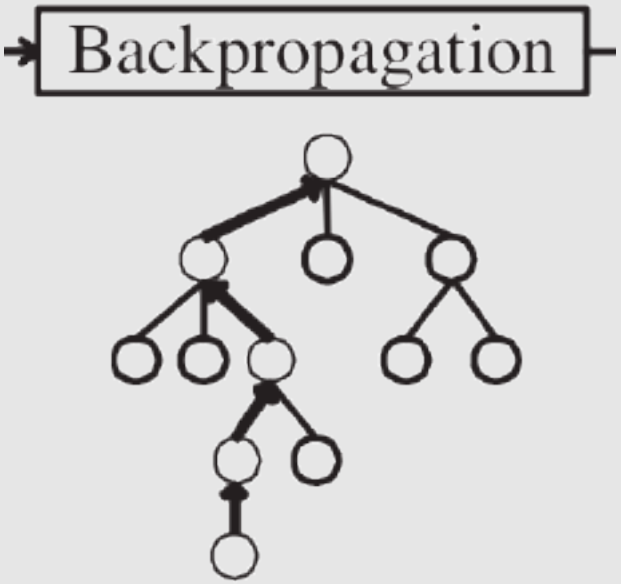
Backpropagation



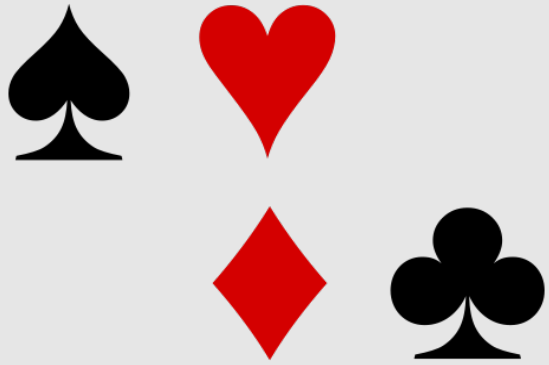
$\hat{V}(P)$ is an estimate of the reward $r(P)$
$\mathrm{T}(P)$ is the number of samples

- Sample-weighted average

$$\hat{V}(n) = \sum_j \frac{\mathrm{T}(c_j)}{\mathrm{T}(n)} \hat{V}(c_j)$$



Backpropagation



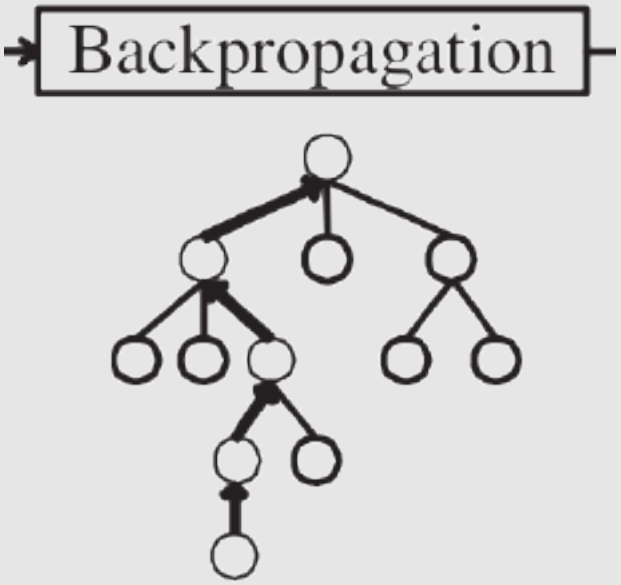
$\hat{V}(P)$ is an estimate of the reward $r(P)$
$\mathrm{T}(P)$ is the number of samples

- Sample-weighted average

$$\hat{V}(n) = \sum_j \frac{\mathrm{T}(c_j)}{\mathrm{T}(n)} \hat{V}(c_j)$$

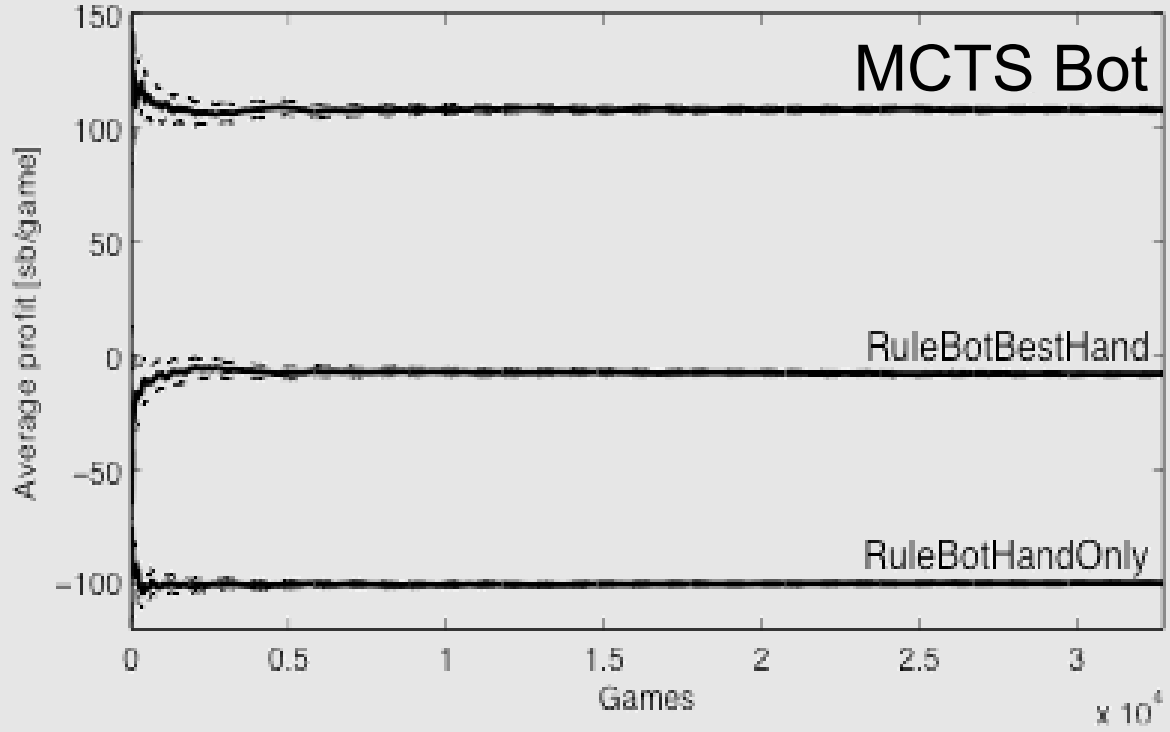
- Maximum child

$$\hat{V}(P) = \max_j \hat{V}(c_j)$$

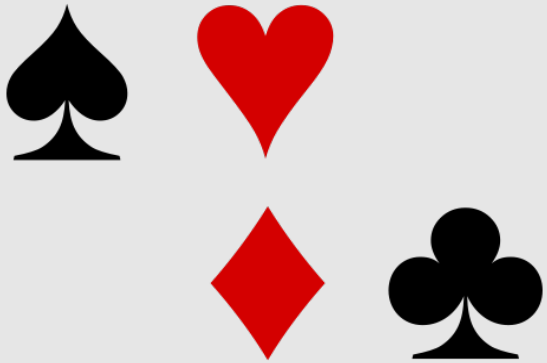


Initial experiments

- 1*MCTS + 2*rule based
- Exploitative!

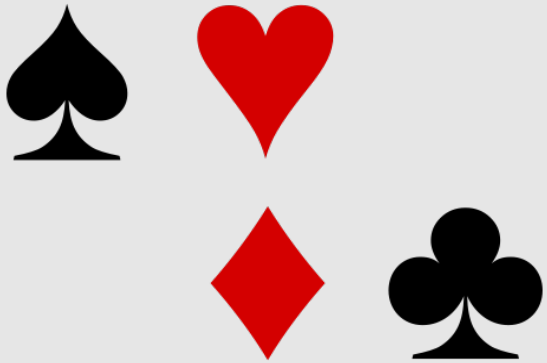


Outline



- Overview approach
  - The Poker game tree
  - Opponent model
  - Monte-Carlo tree search
- Research challenges
  - Search
  - Opponent model
- Conclusion

Outline



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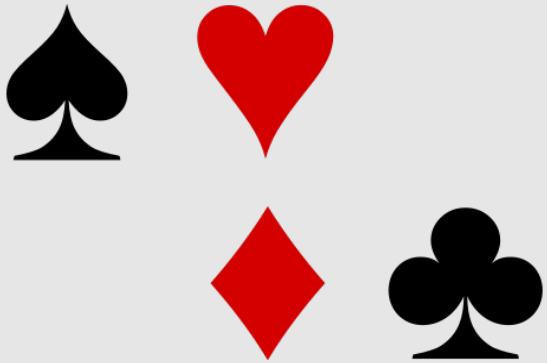

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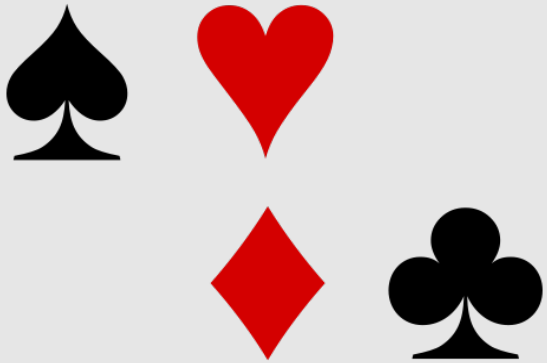

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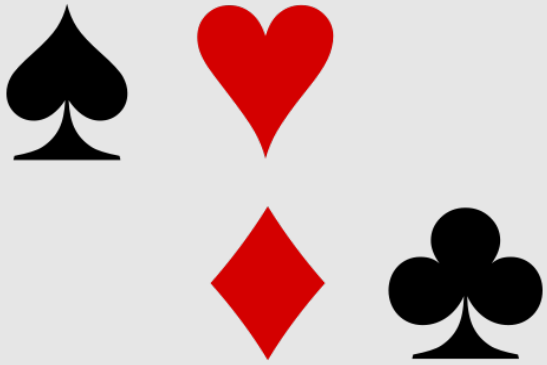
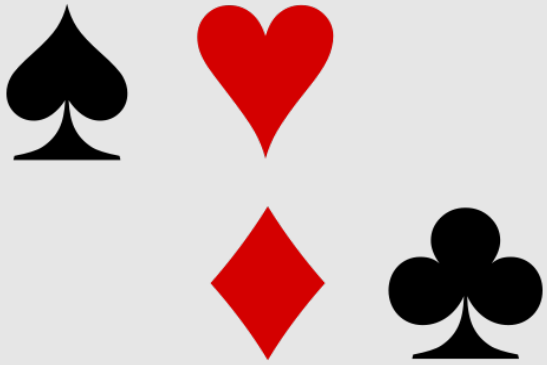
MCTS for games with uncertainty?



- Expected reward distributions (ERD)
- Sample selection using ERD
- Backpropagation of ERD

[VandenBroeck09]

Expected reward distribution



MiniMax

Estimating $r(P)$

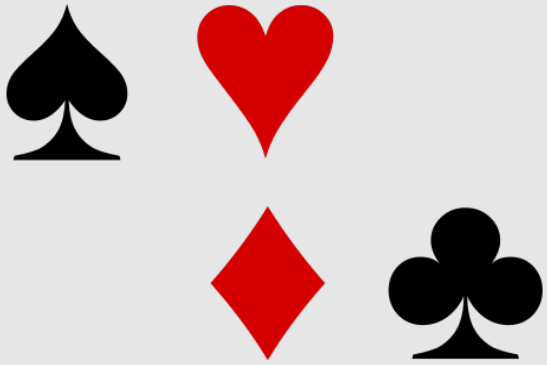
10 samples

100 samples

∞ samples

Variance

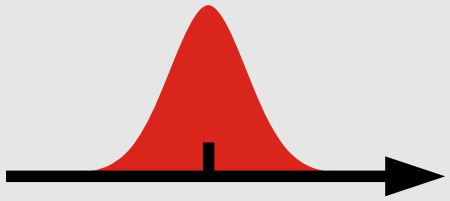
Expected reward distribution



MiniMax

Estimating $r(P)$

10 samples

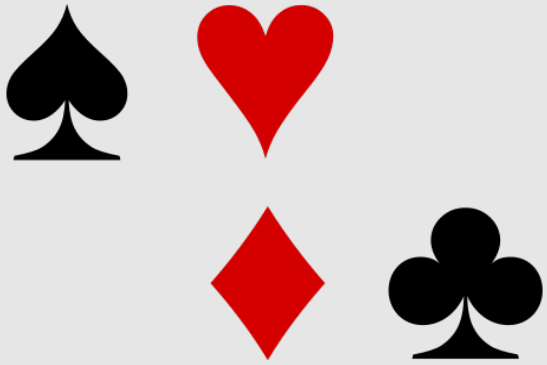


100 samples

∞ samples

Variance

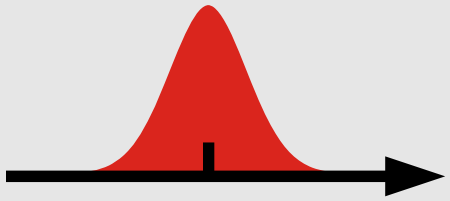
Expected reward distribution



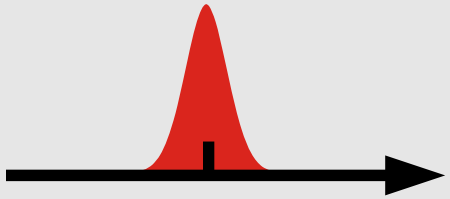
MiniMax

Estimating $r(P)$

10 samples



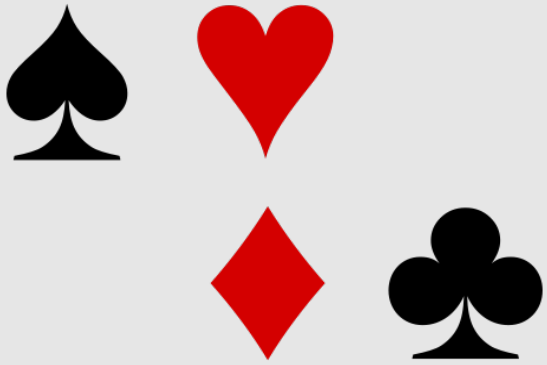
100 samples



∞ samples

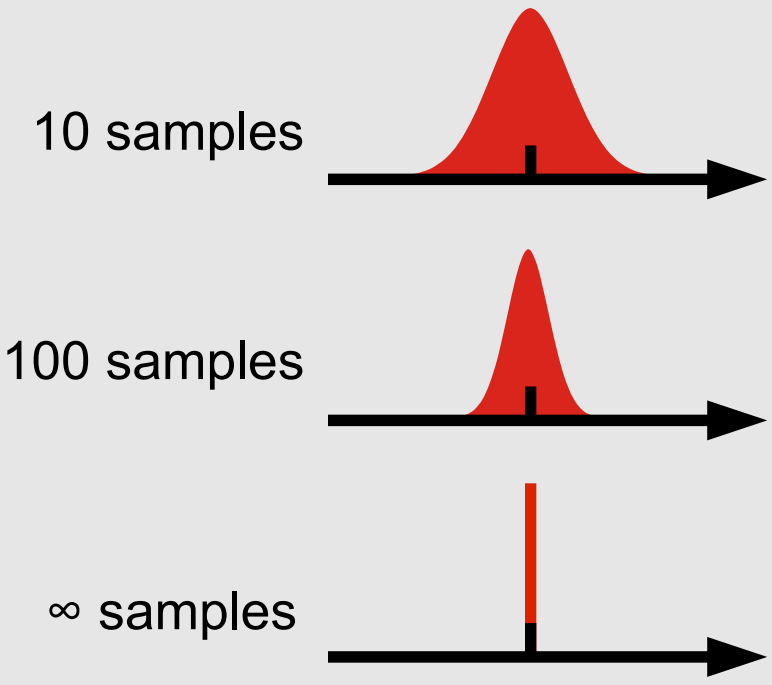
Variance

Expected reward distribution

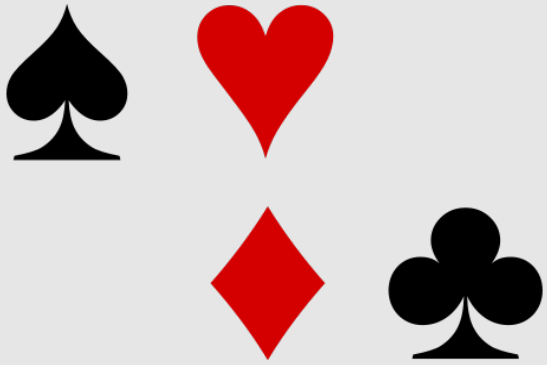


MiniMax

Estimating $r(P)$

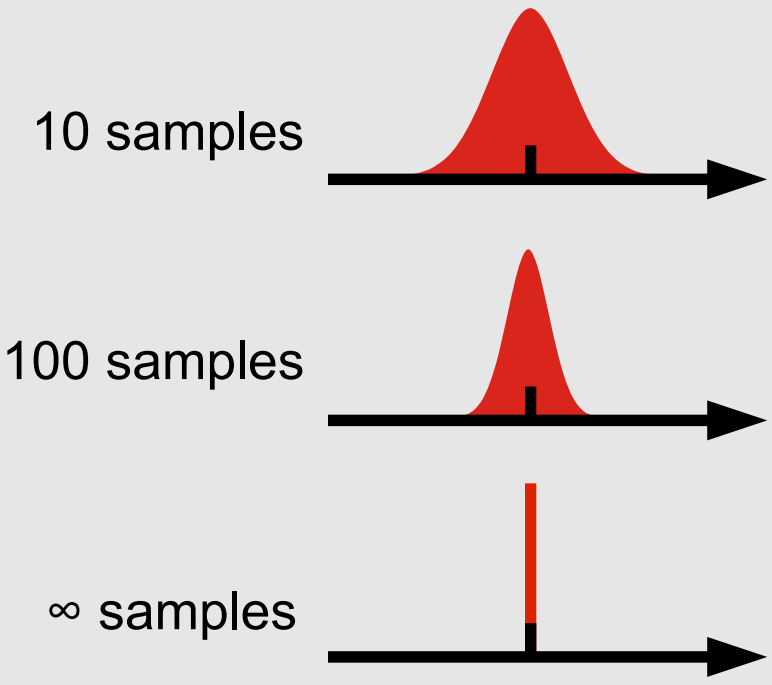


Expected reward distribution



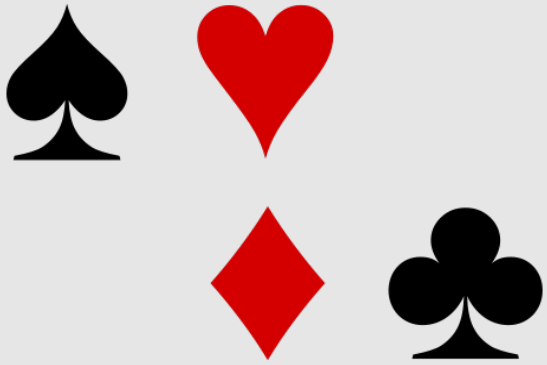
MiniMax

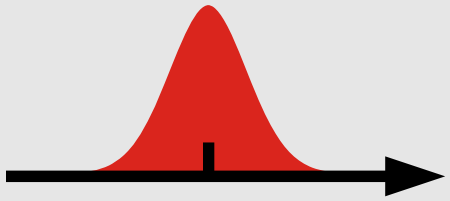
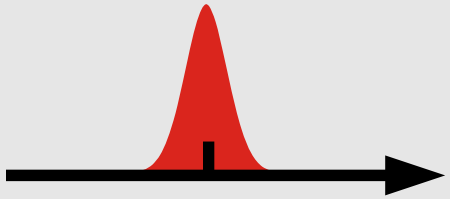
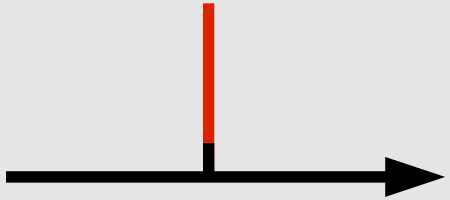
Estimating $r(P)$



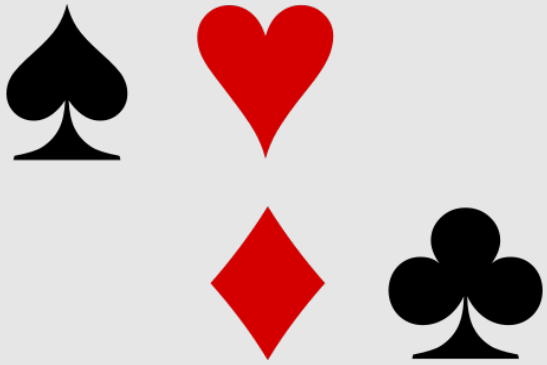
Variance *Sampling*

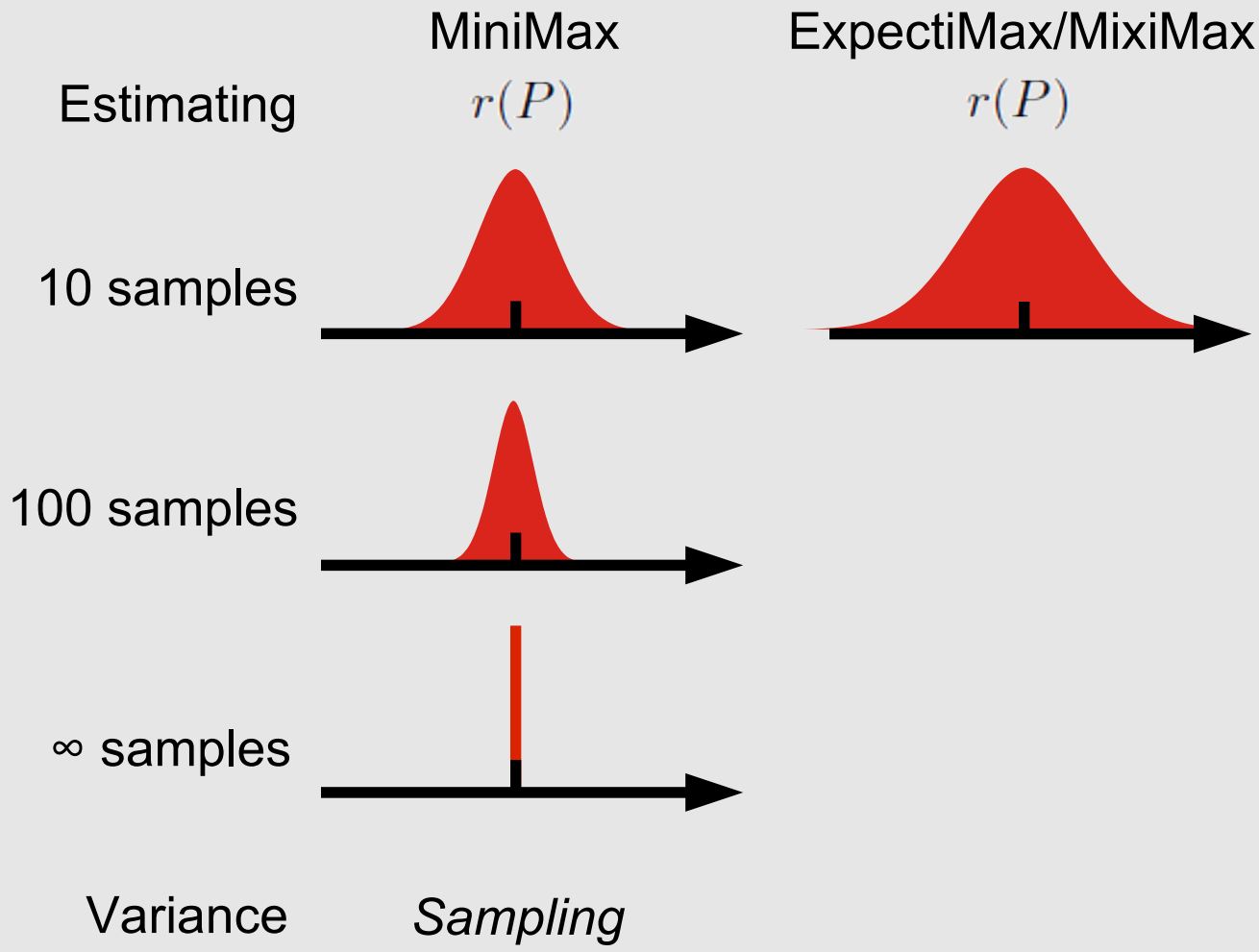
Expected reward distribution



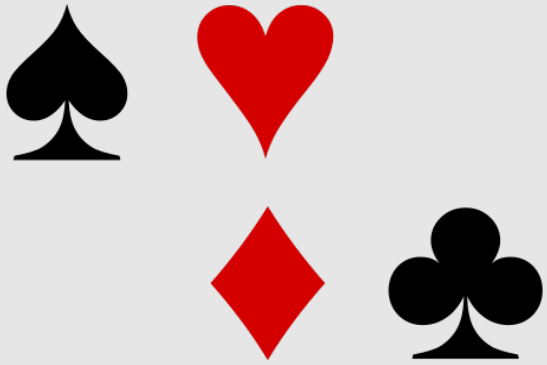
	MiniMax	ExpectiMax/MixiMax
Estimating	$r(P)$	$r(P)$
10 samples		
100 samples		
∞ samples		
Variance	*Sampling*	

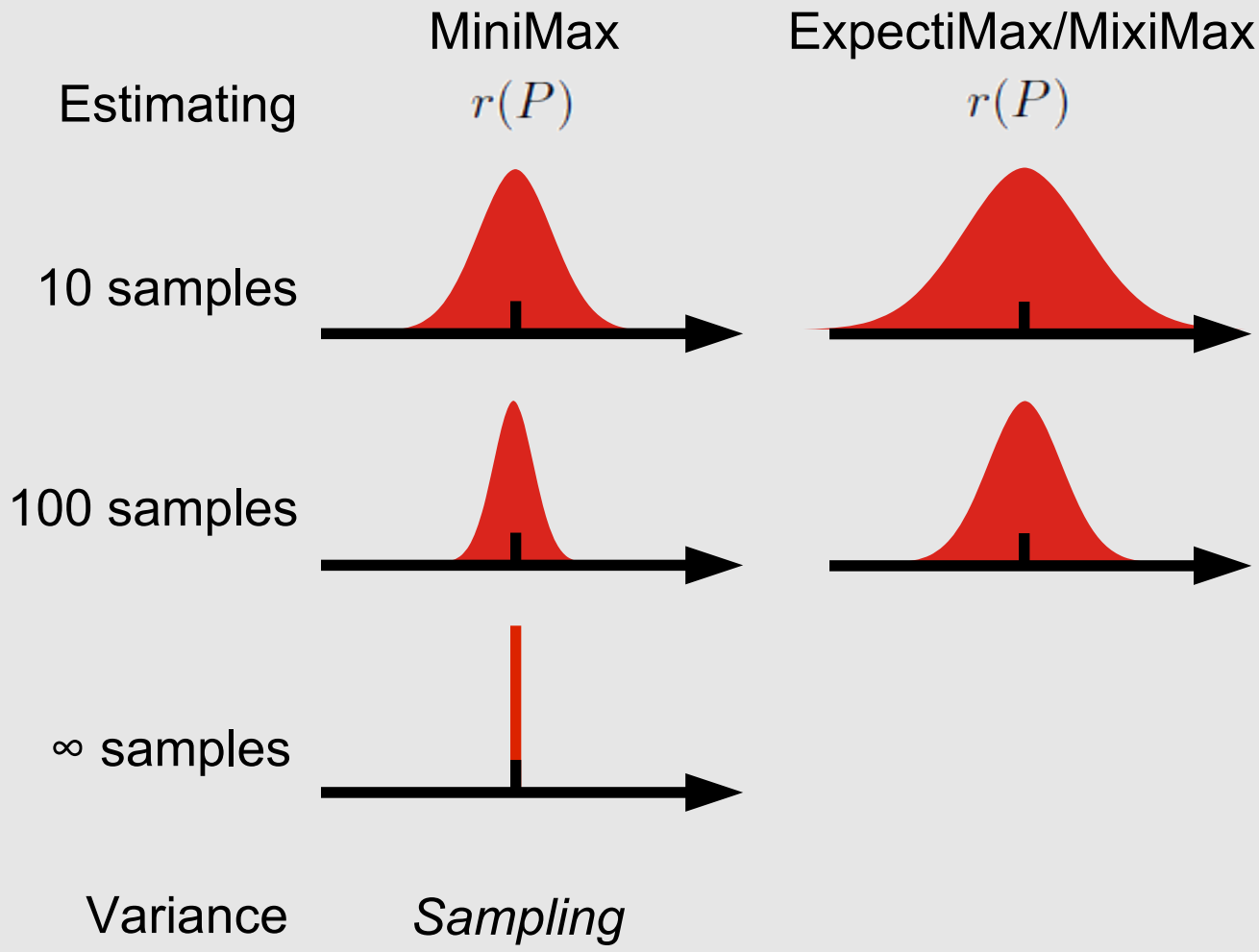
Expected reward distribution



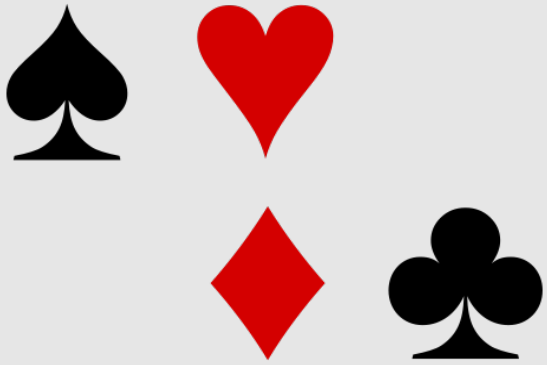


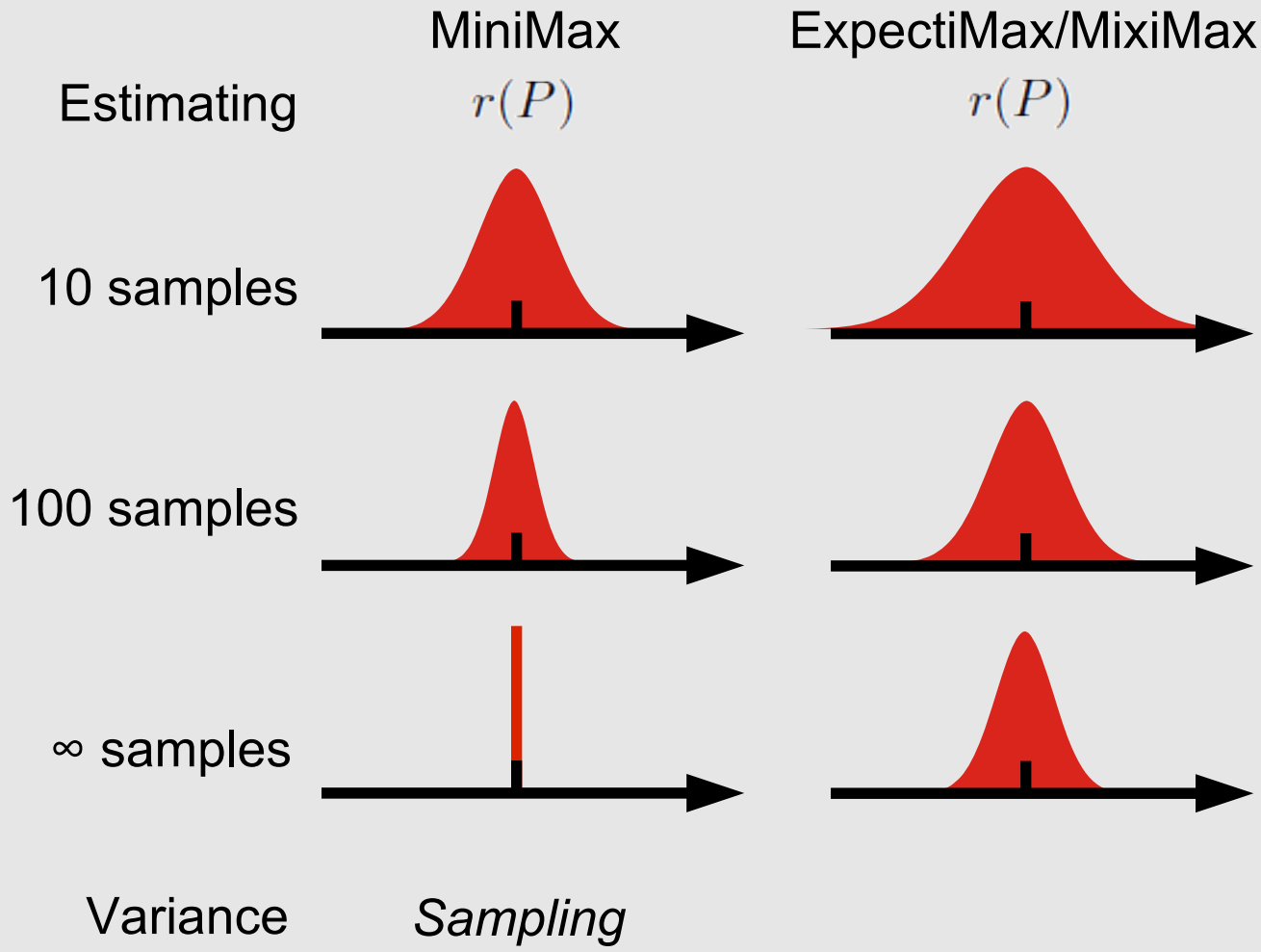
Expected reward distribution



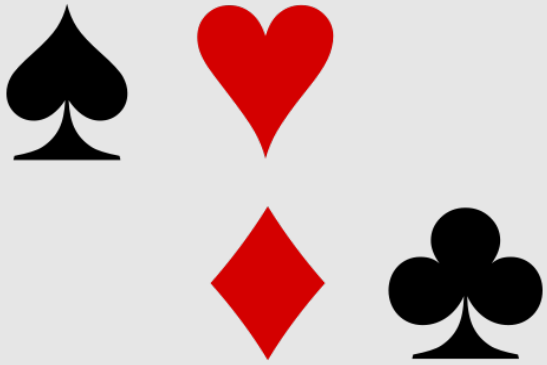


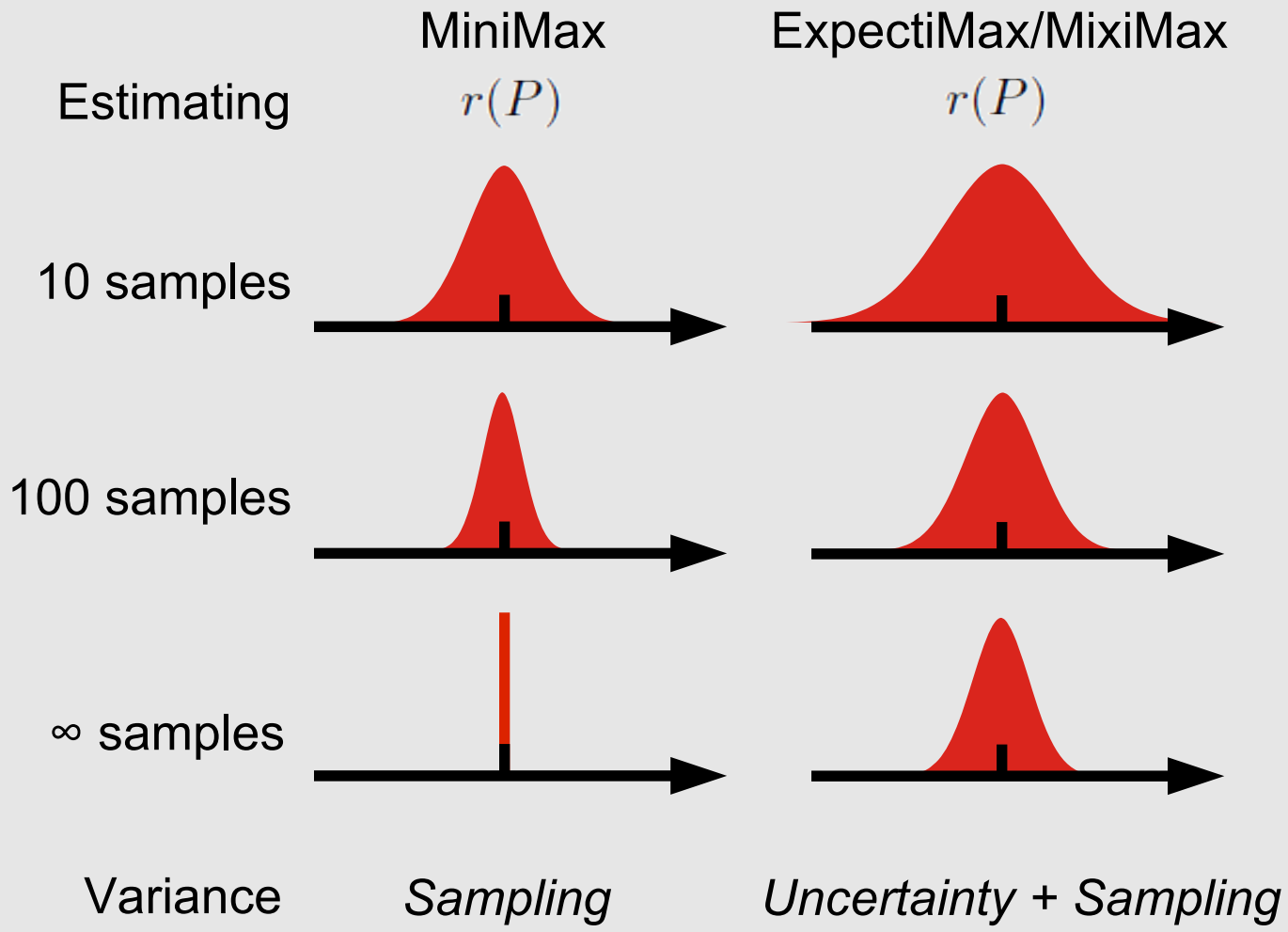
Expected reward distribution



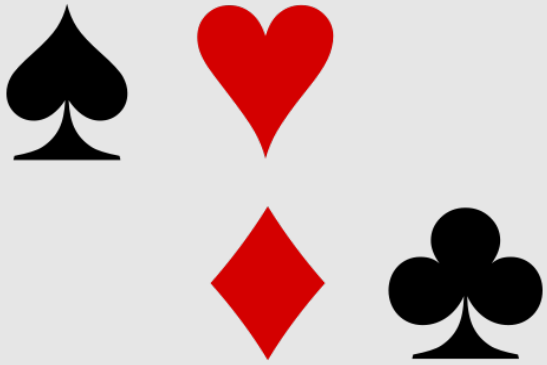


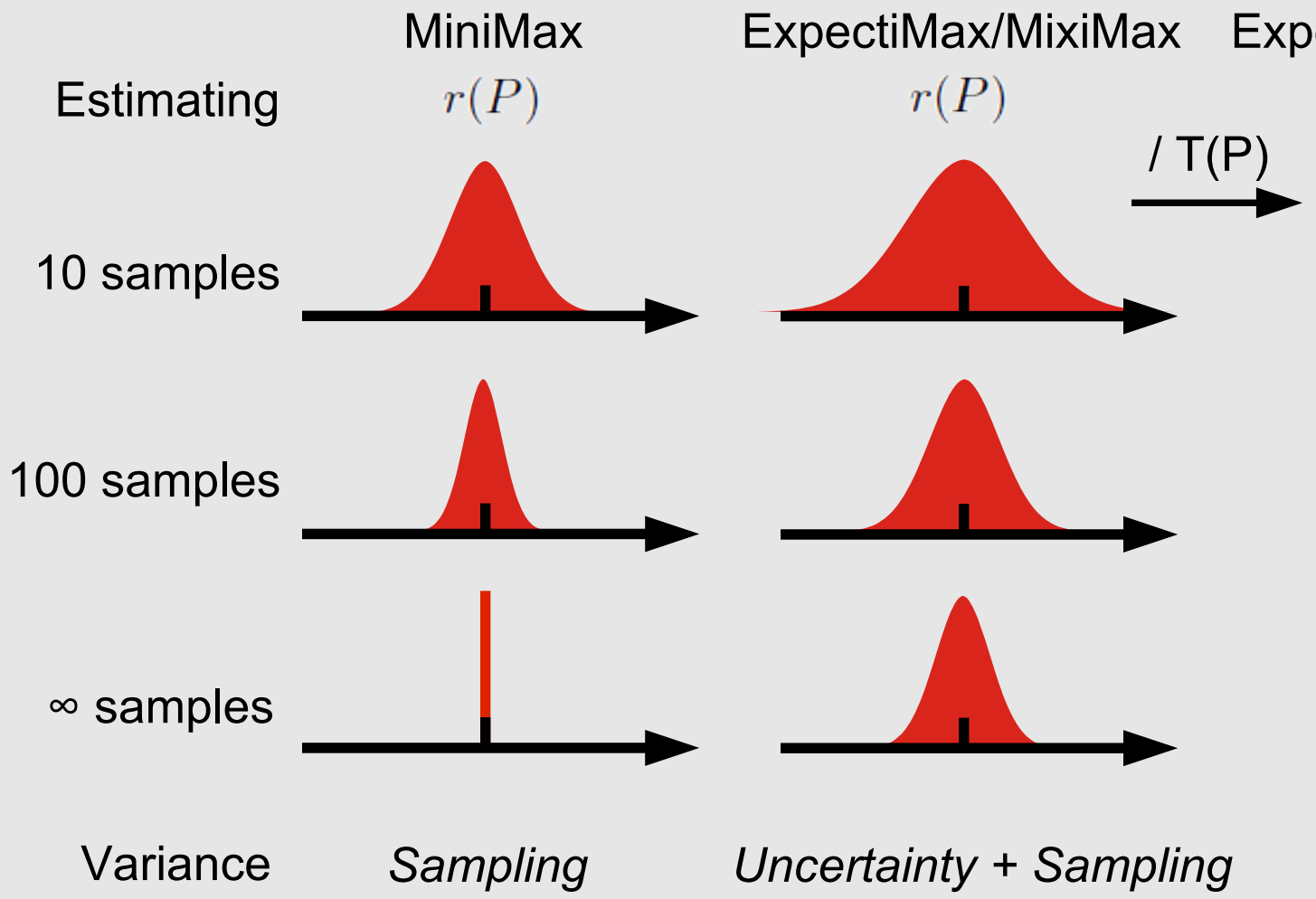
Expected reward distribution



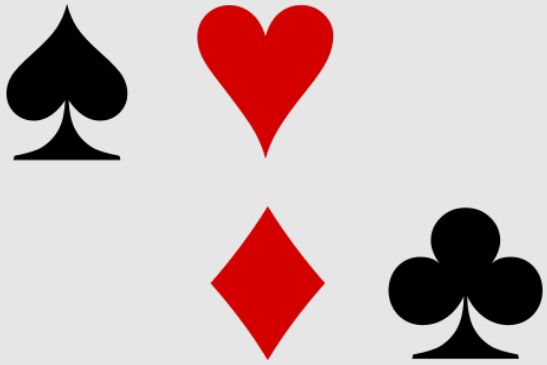


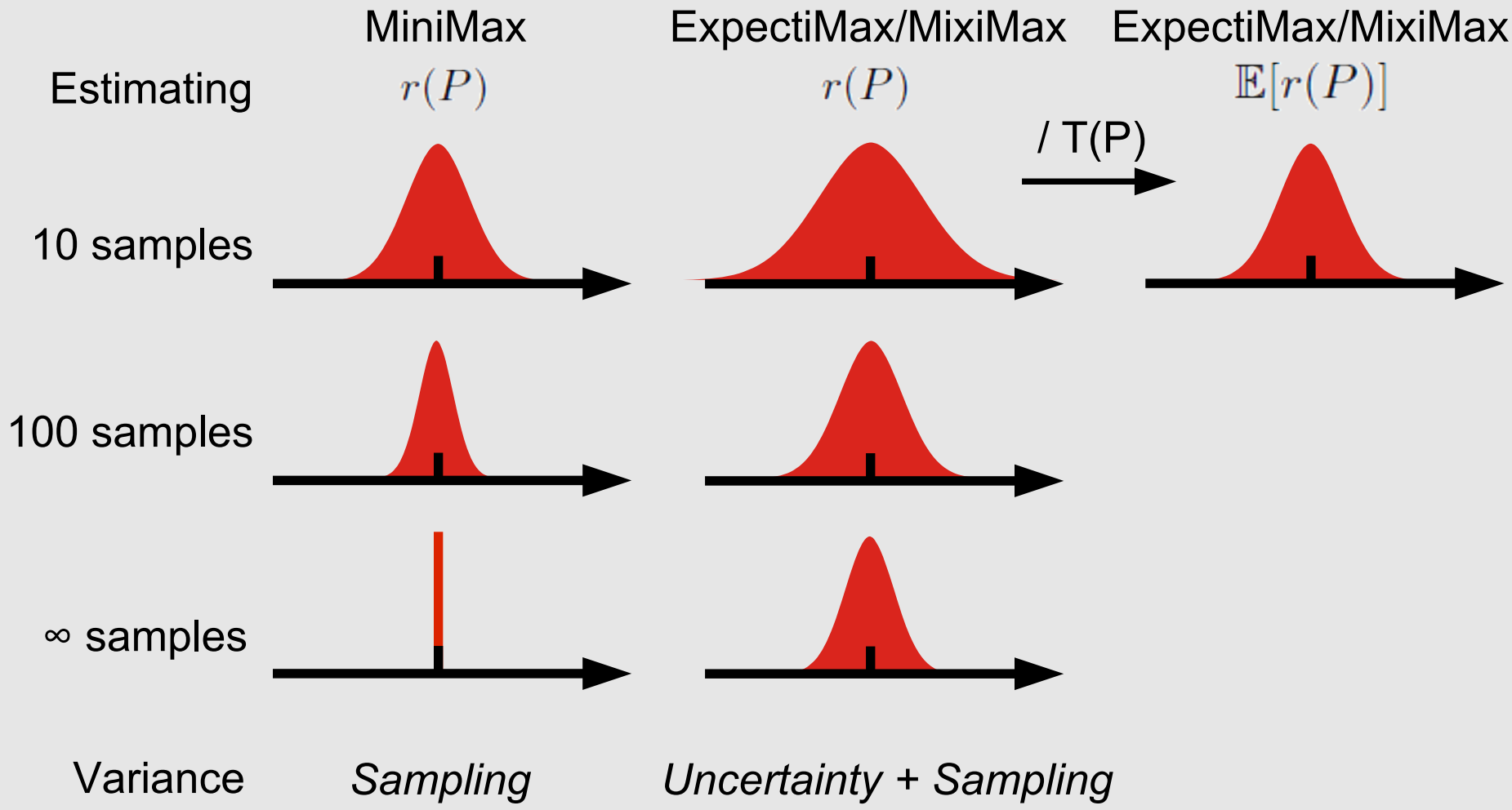
Expected reward distribution



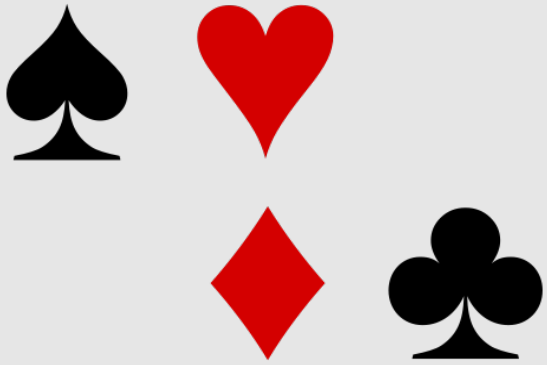


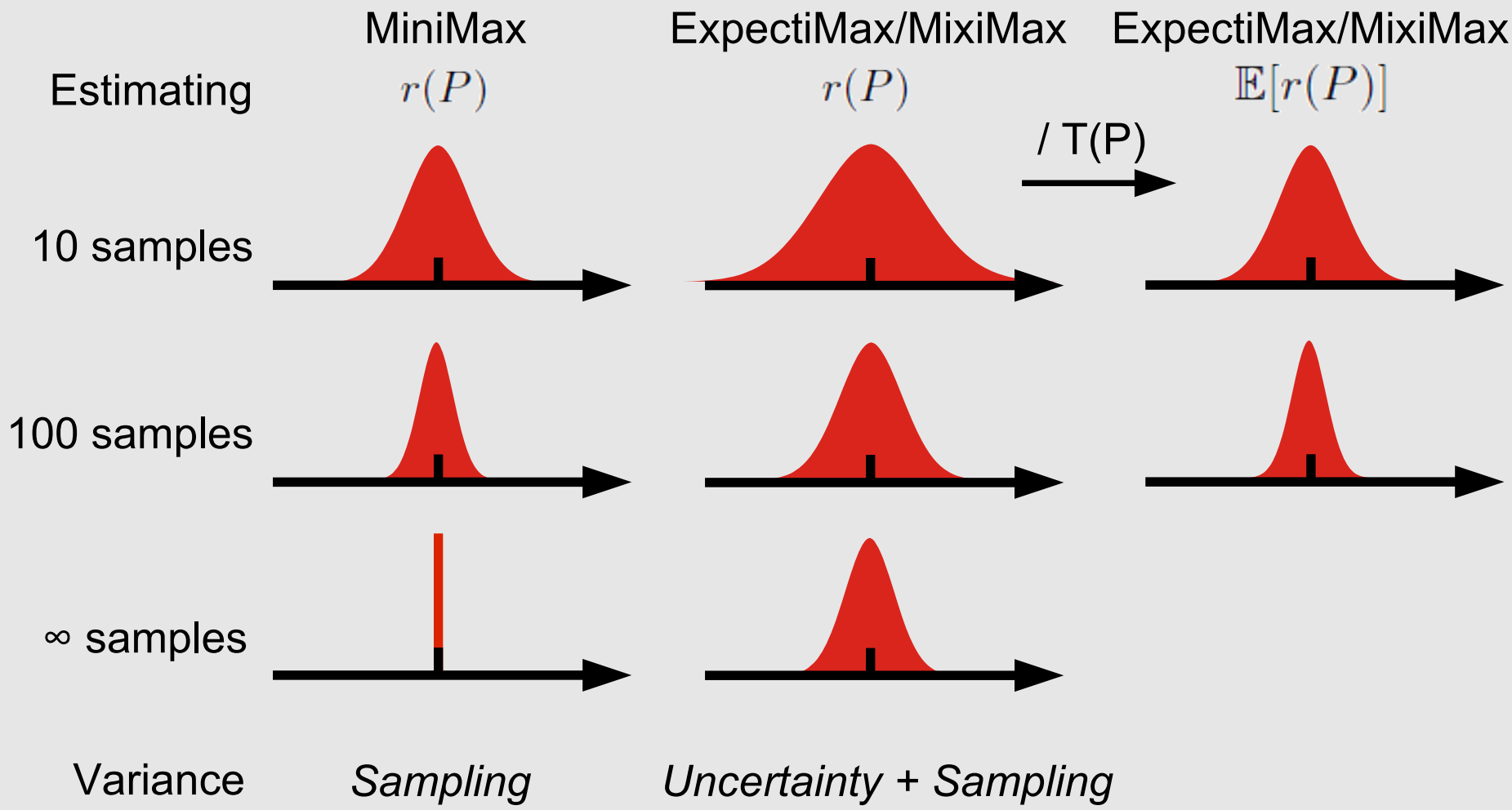
Expected reward distribution





Expected reward distribution





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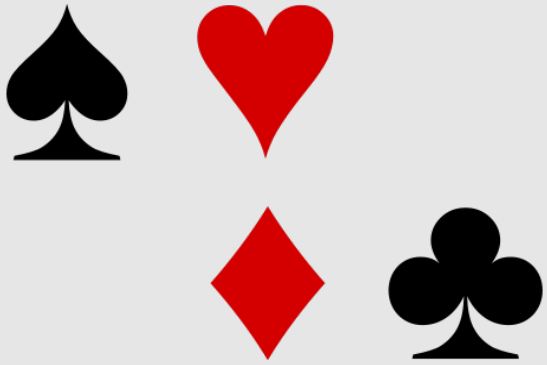

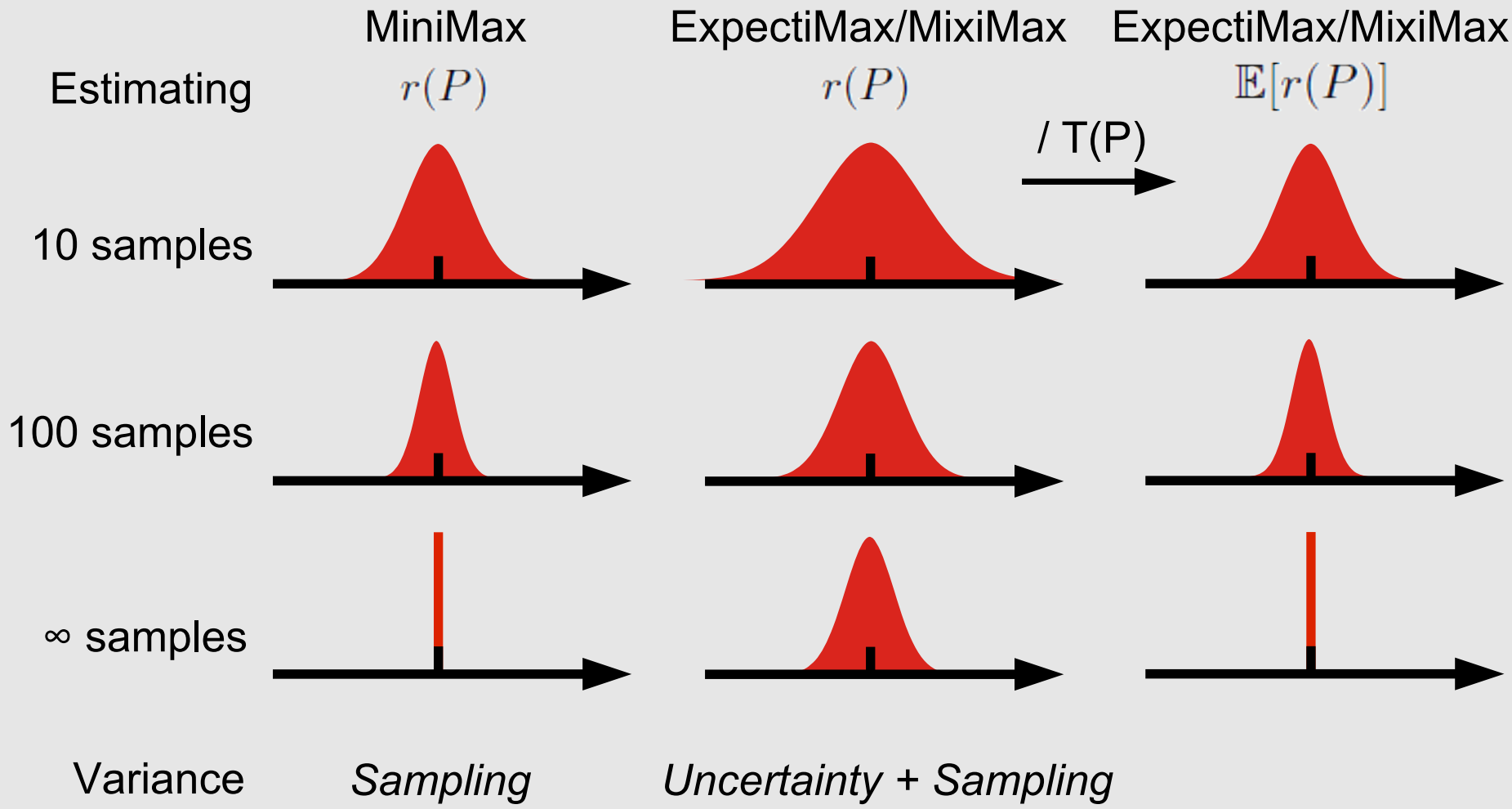

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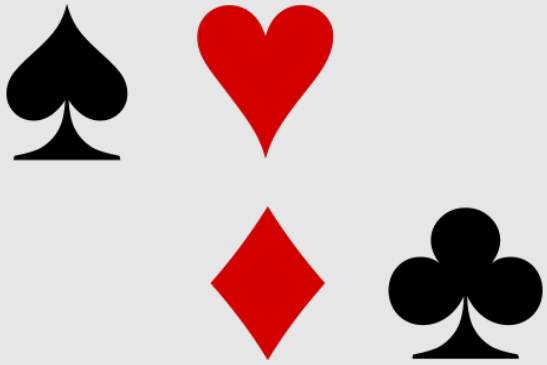

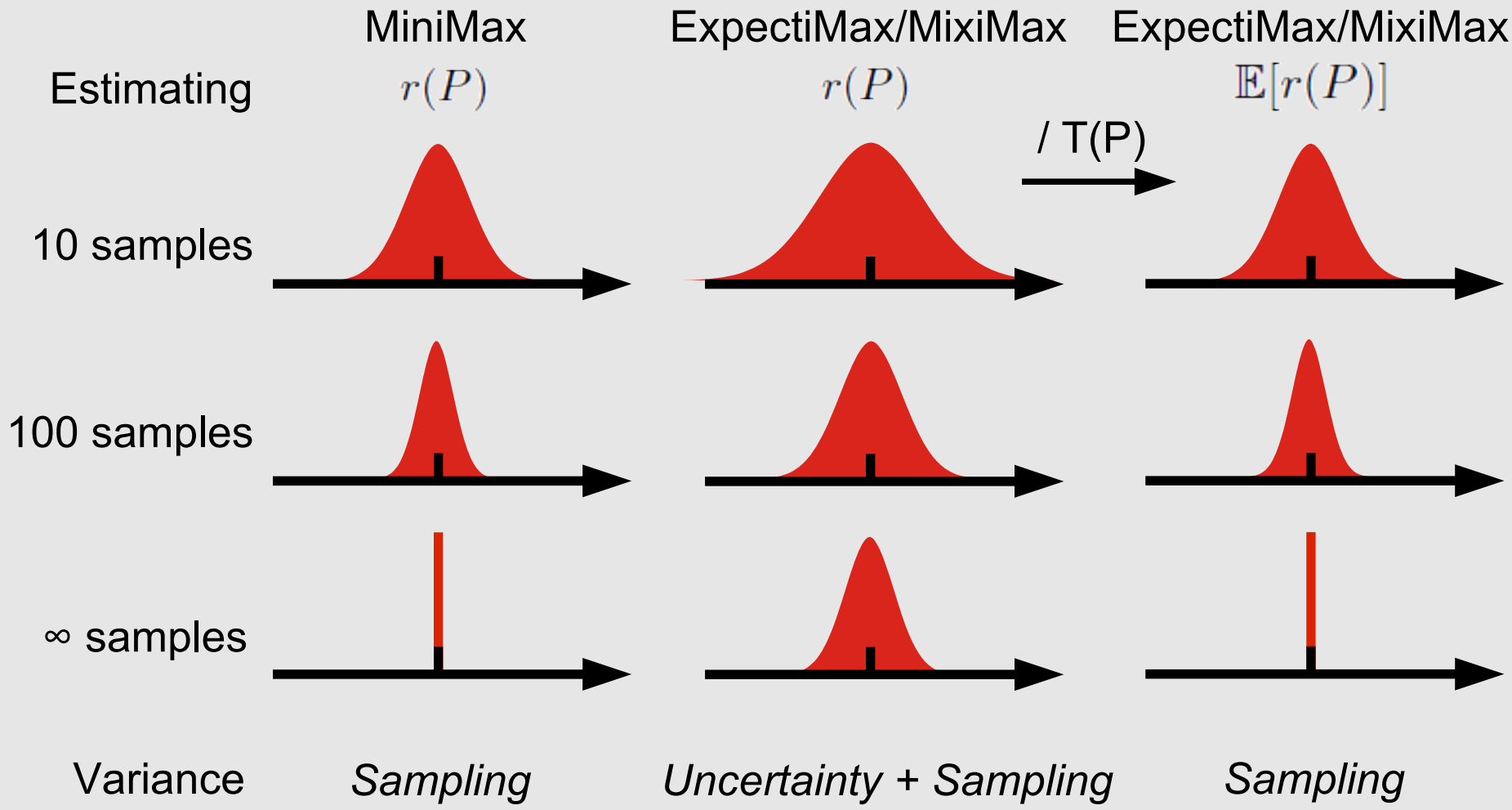

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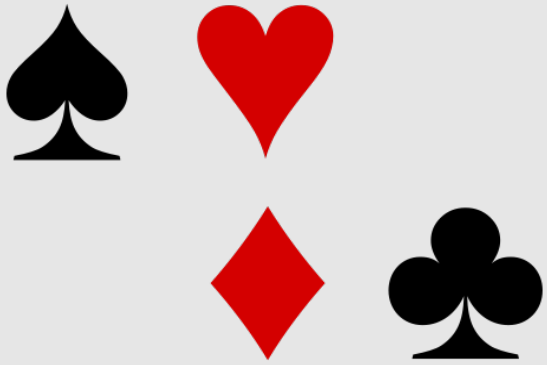
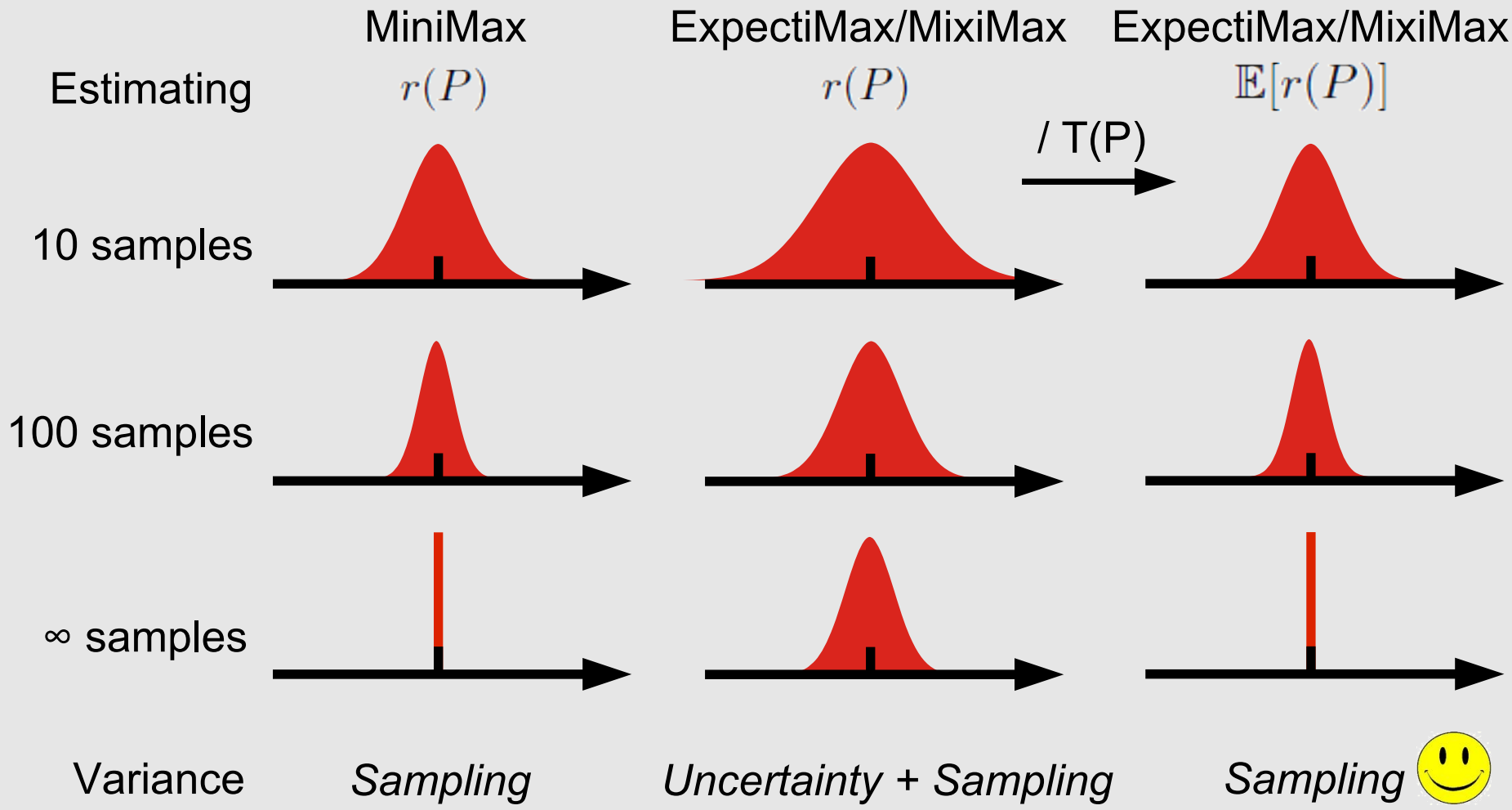
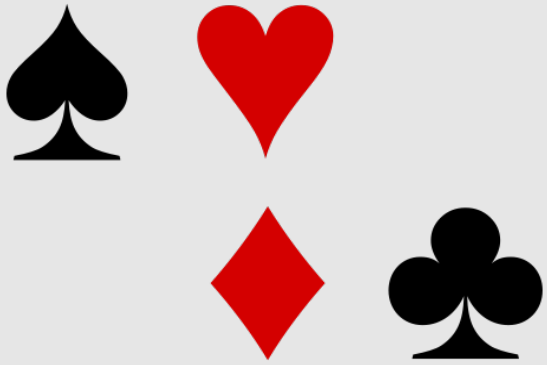




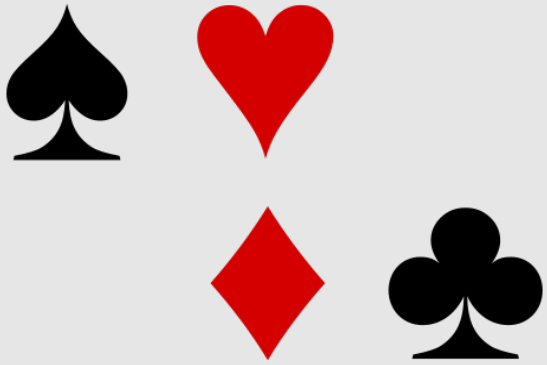
ERD selection strategy



- Objective?
  - Find maximum expected reward
  - Sample more in subtrees with
    - (1) High expected reward
    - (2) Uncertain estimate
- UCT does (1) but not really (2)
- CrazyStone does (1) and (2) for deterministic games (Go)
- **UCT+ selection**: $\hat{V}(c_i) + C.\sigma_{\hat{V},c_i}$

  (1) (2)

ERD selection strategy



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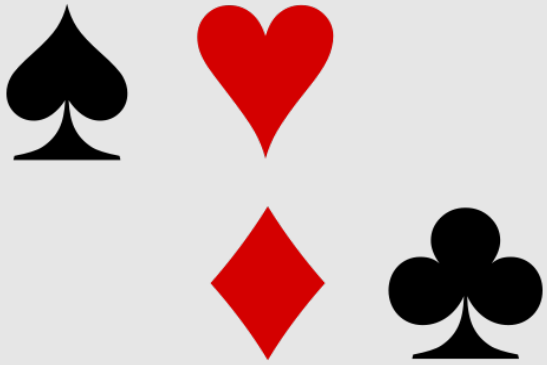
"Expected value under perfect play"

ERD selection strategy



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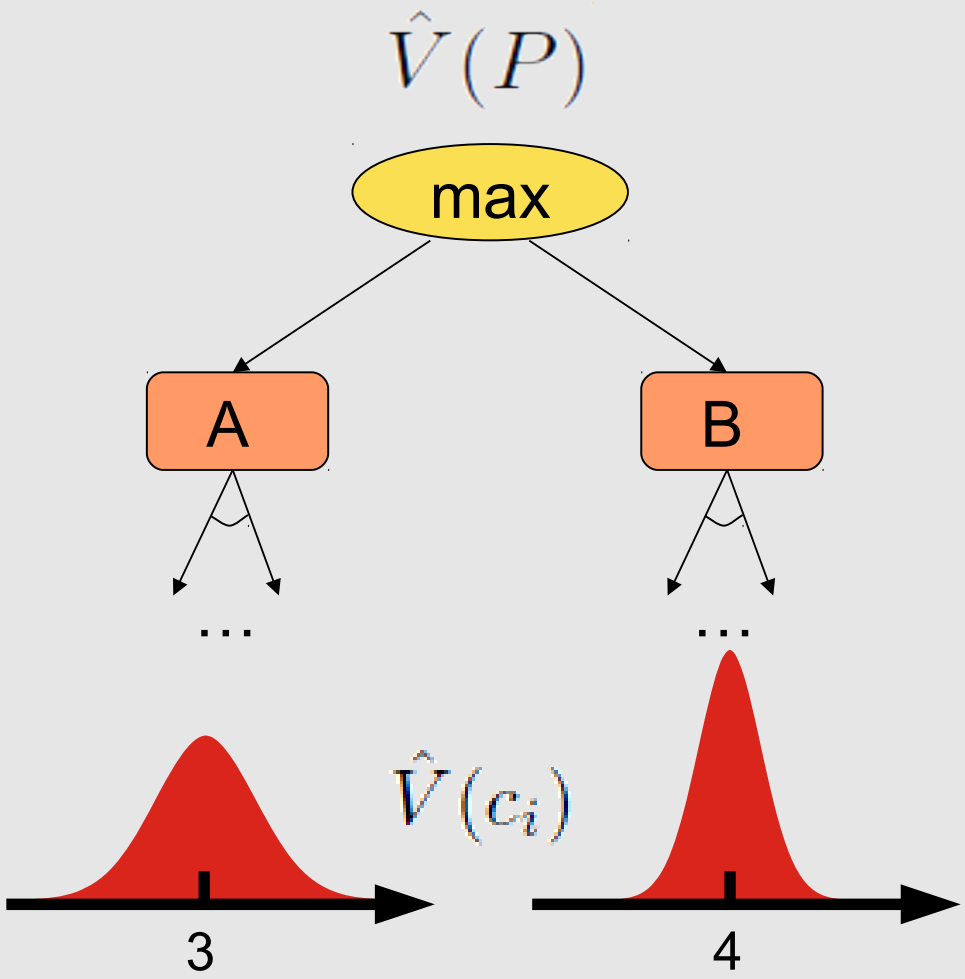
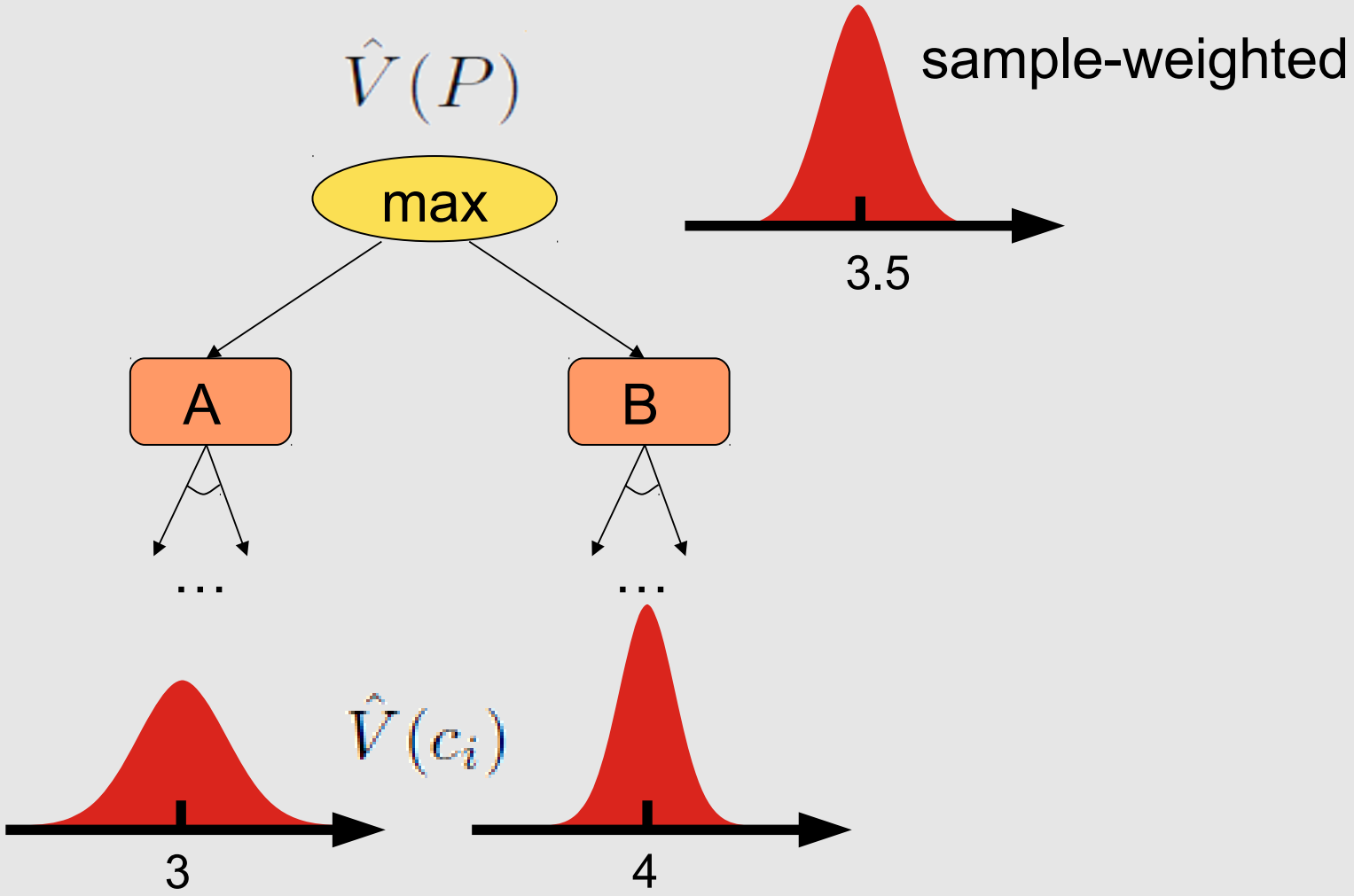
"Measure of uncertainty due to sampling"

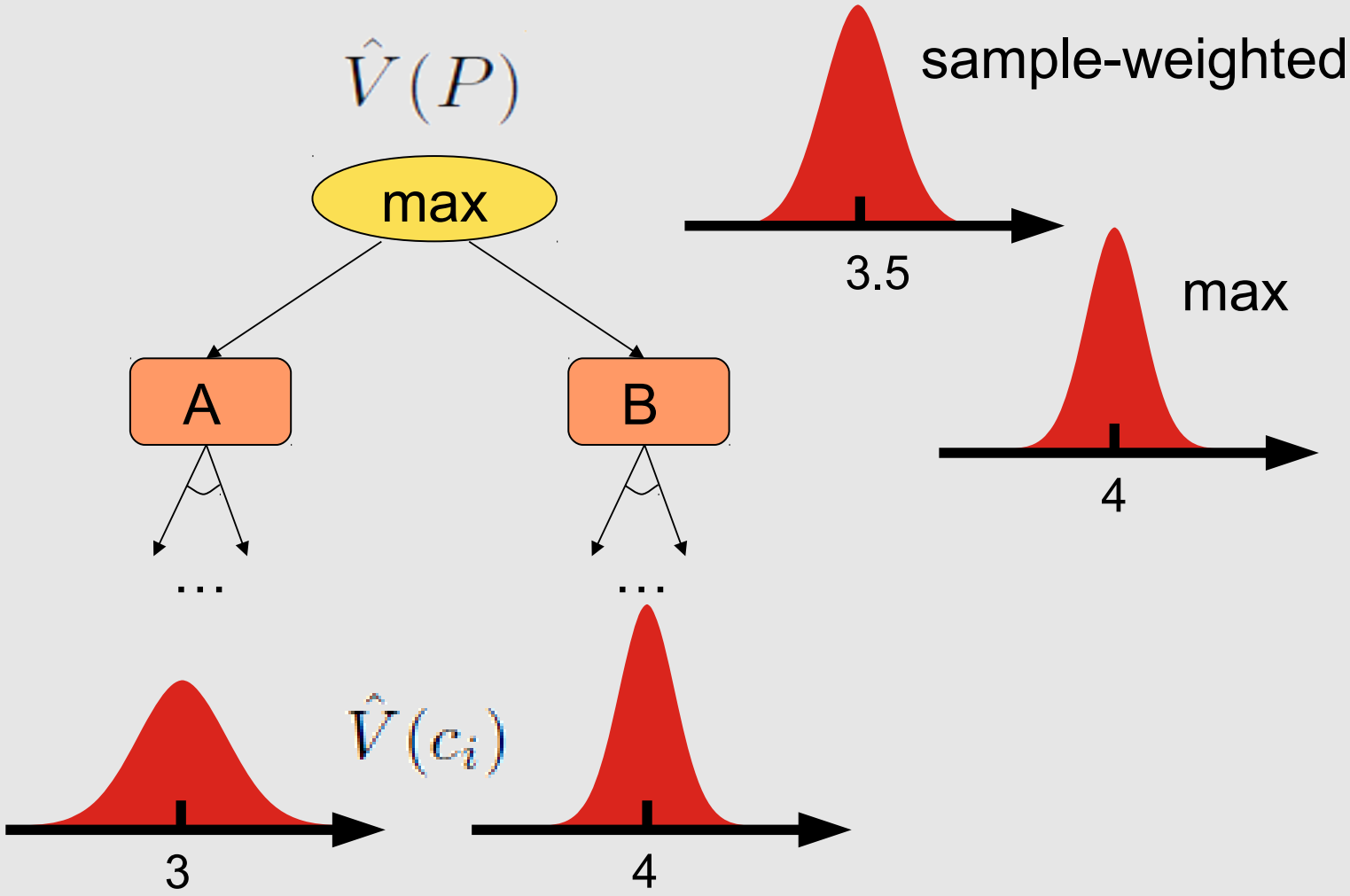
ERD max-distribution backpropagation



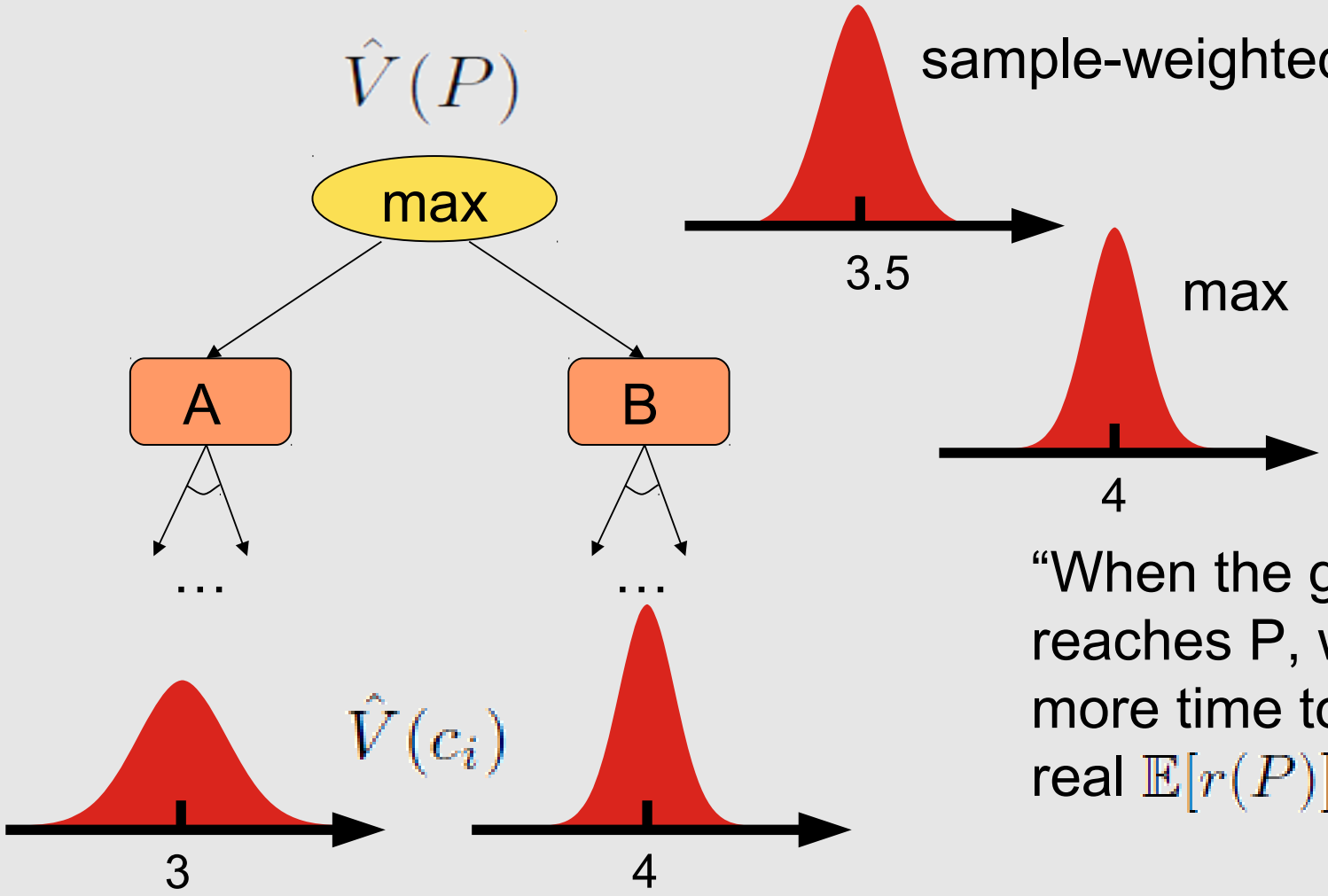
ERD max-distribution backpropagation



ERD max-distribution backpropagation

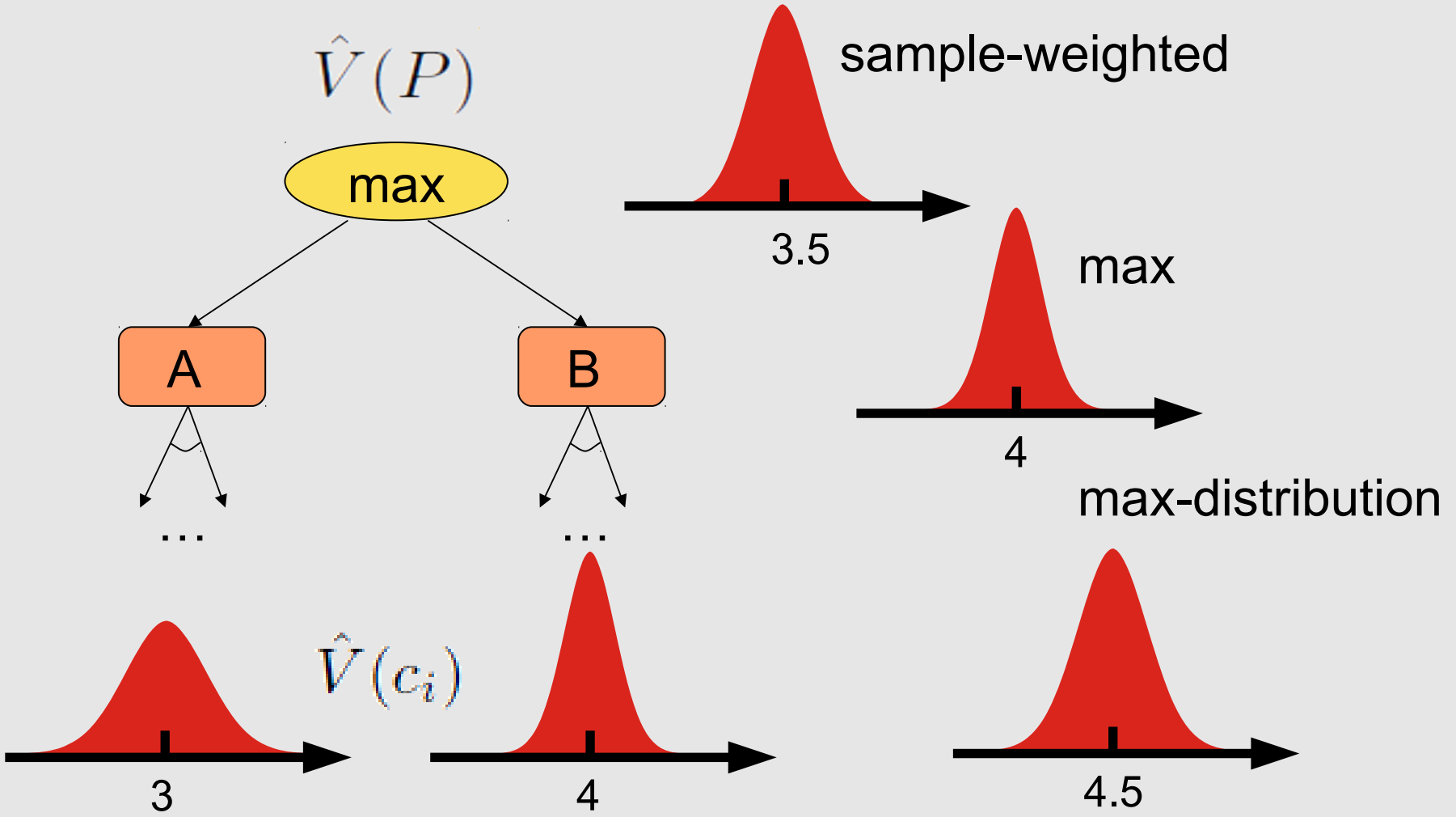


ERD max-distribution backpropagation

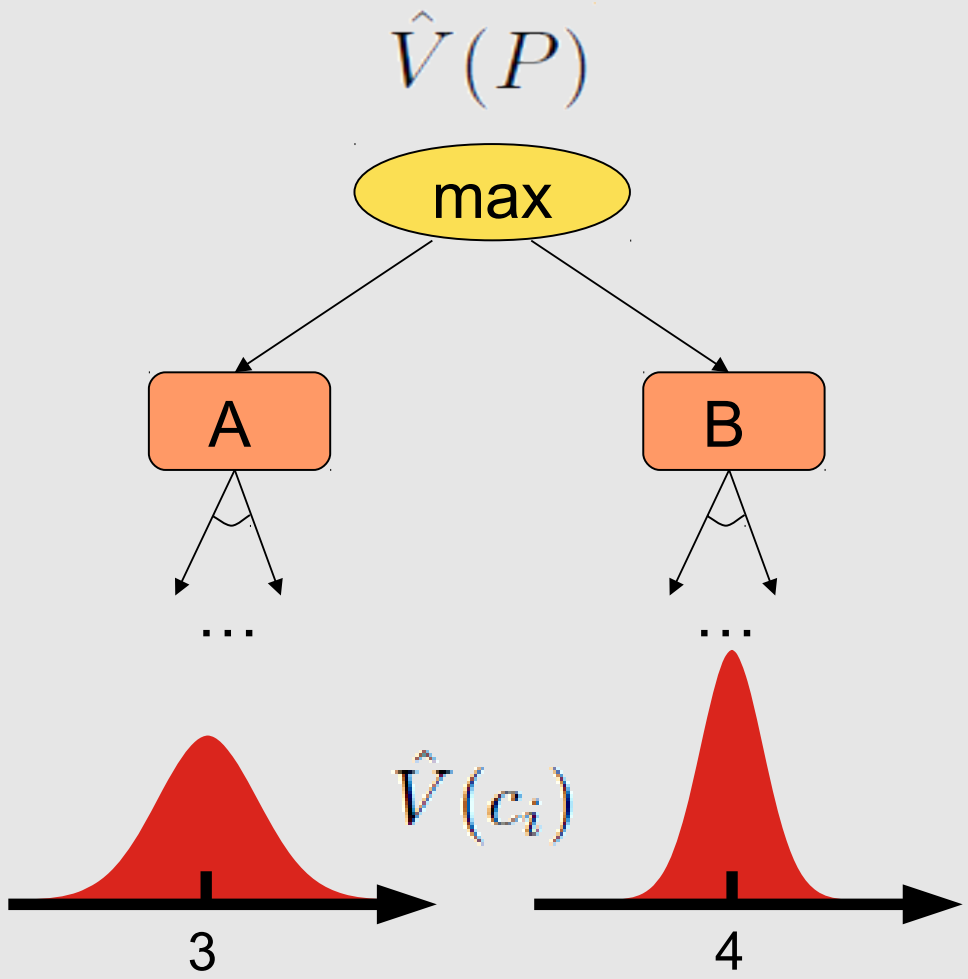


"When the game reaches P, we'll have more time to find the real $\mathbb{E}[r(P)]$ "

ERD max-distribution backpropagation



ERD max-distribution backpropagation

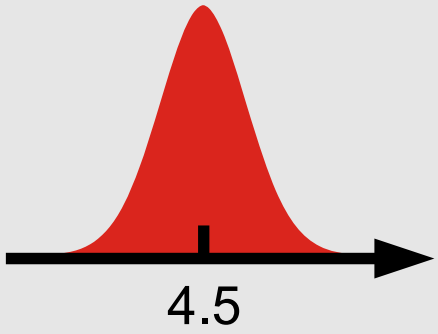


P(B<4) = 0.5 P(B>4) = 0.5

P(A<4) = 0.8 P(A>4) = 0.2

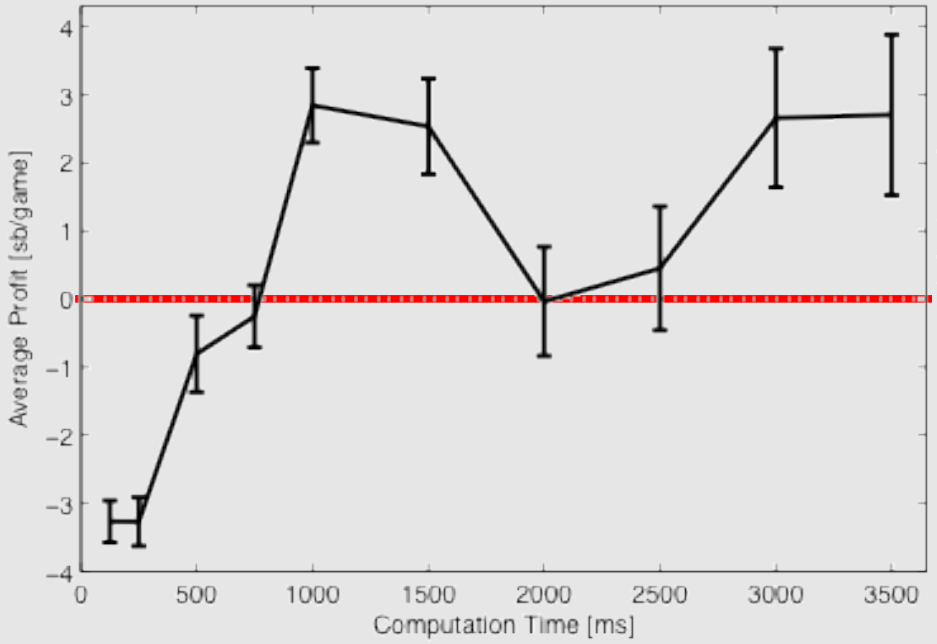
	A<4	A>4
B<4	0.8*0.5	0.2*0.5
B>4	0.8*0.5	0.2*0.5

P(max(A,B)>4) = 0.6
> 0.5

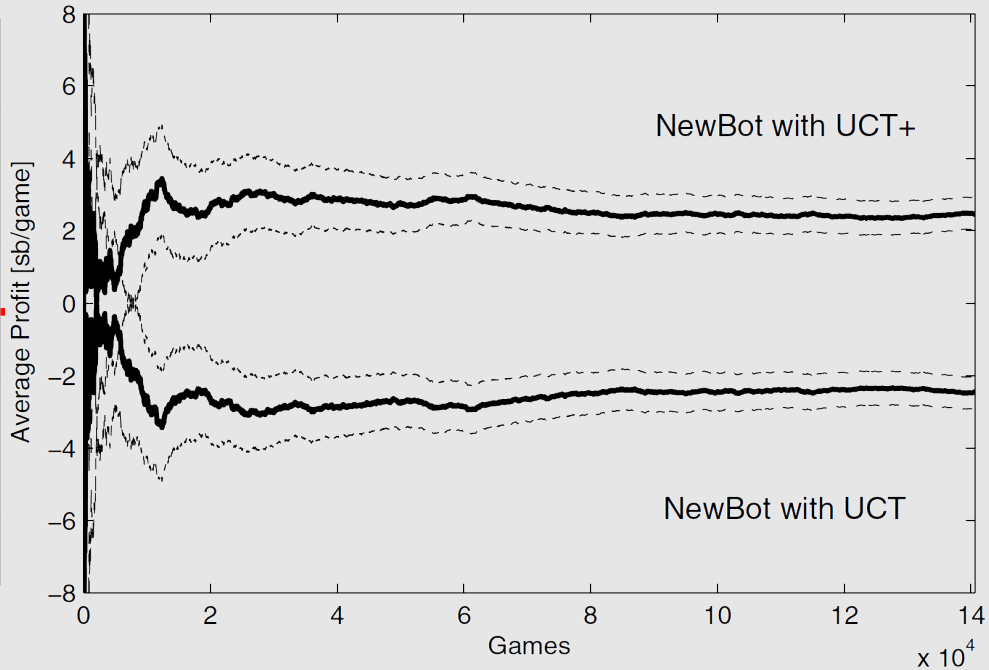


Experiments

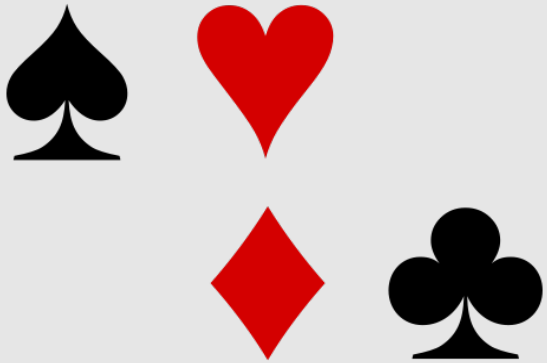
- 2*MCTS
  - Max-distribution
  - Sample-weighted



- 2*MCTS
  - UCT+ (stddev)
  - UCT



Outline



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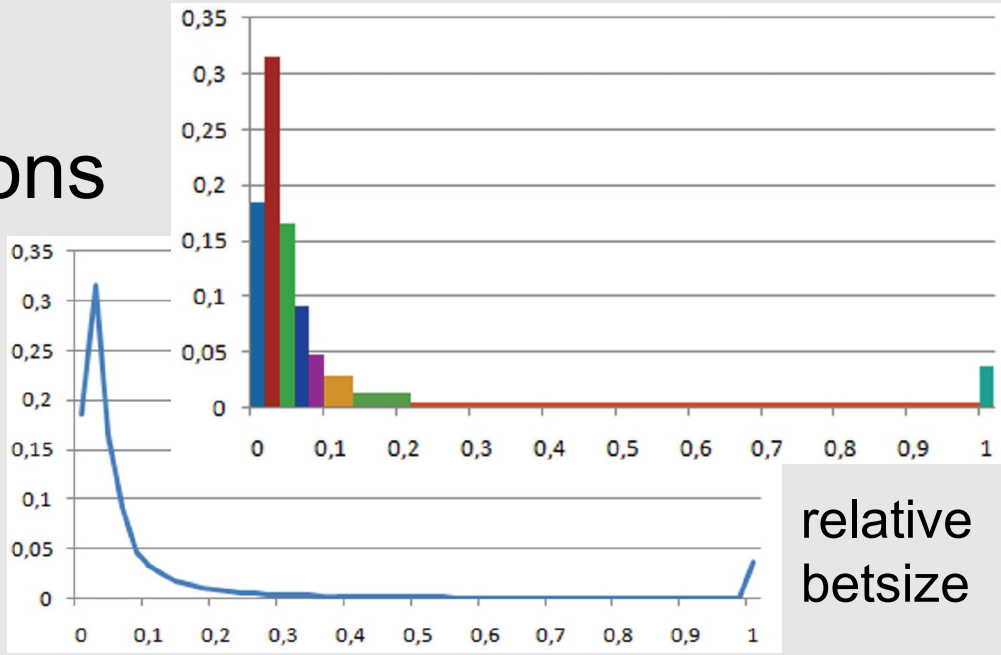
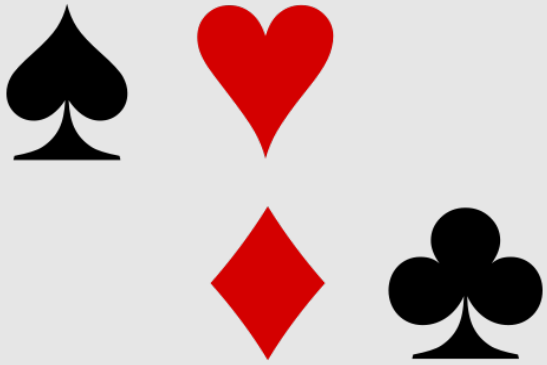
- Conclusion

Dealing with continuous actions



- Sample discrete actions
- Progressive unpruning [Chaslot08]
  (ignores smoothness of EV function)
- ...
- Tree learning search (work in progress)

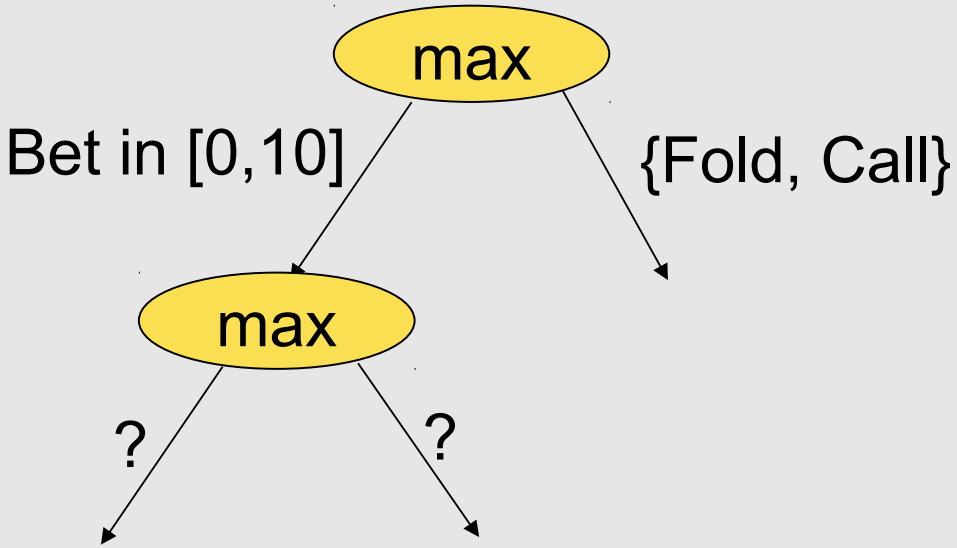
Tree learning search



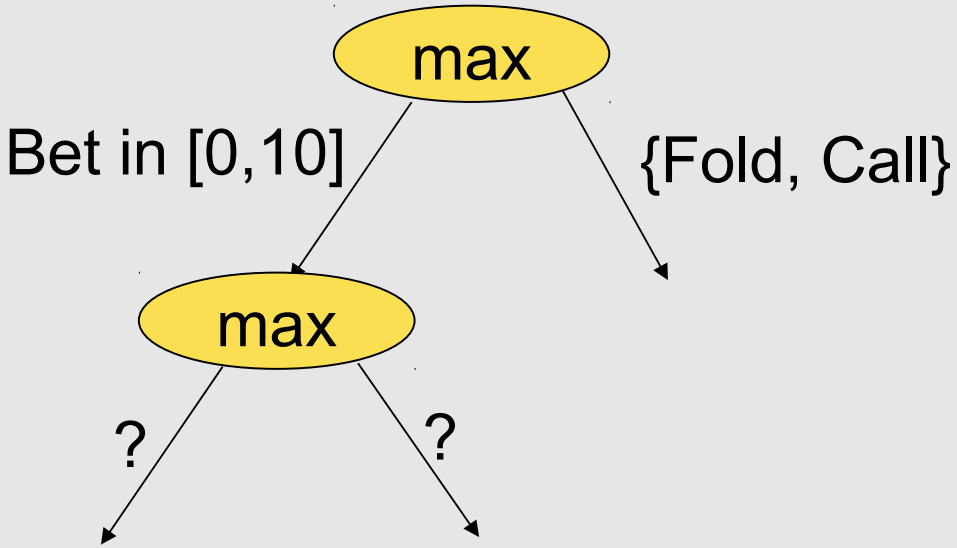
- Based on regression tree induction from ***data streams***
  - training examples arrive ***quickly***
  - nodes ***split*** when significant reduction in stddev
  - training examples are immediately ***forgotten***

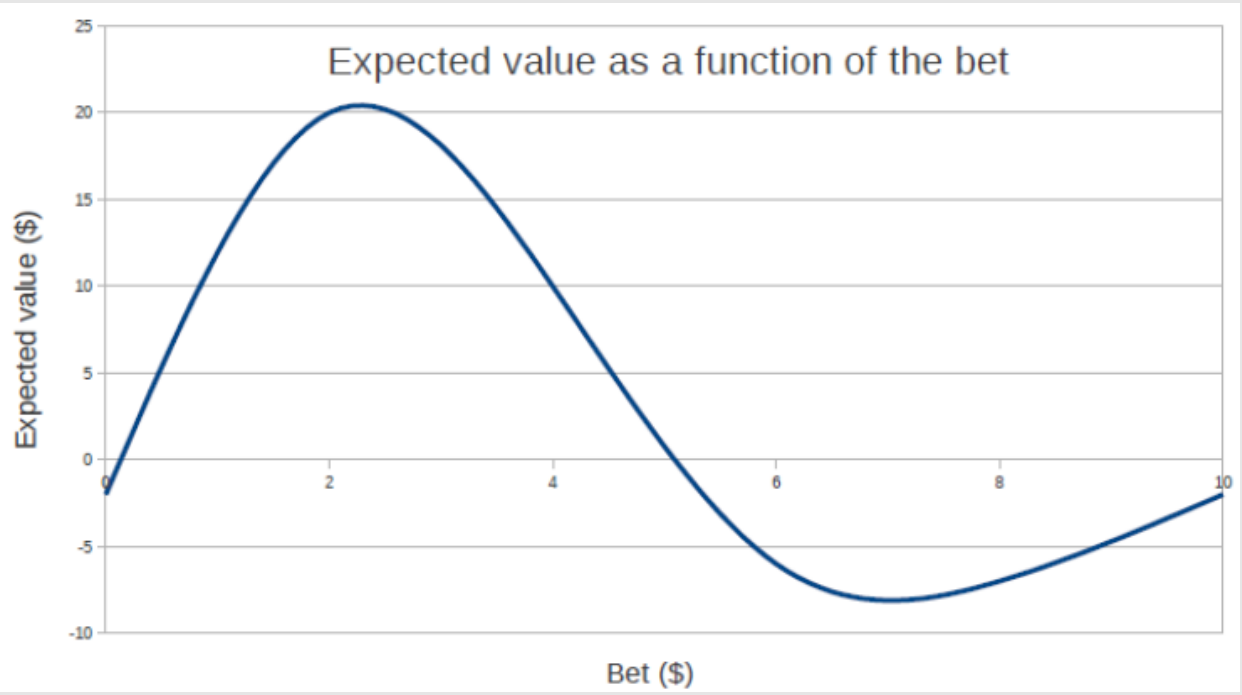
- Edges in TLS tree are not actions, but ***sets of actions***, e.g., (raise in [2,40]), (fold or call)
- MCTS provides a ***stream*** of (action,EV) examples
- Split action sets to reduce stddev of EV (when significant)

Tree learning search

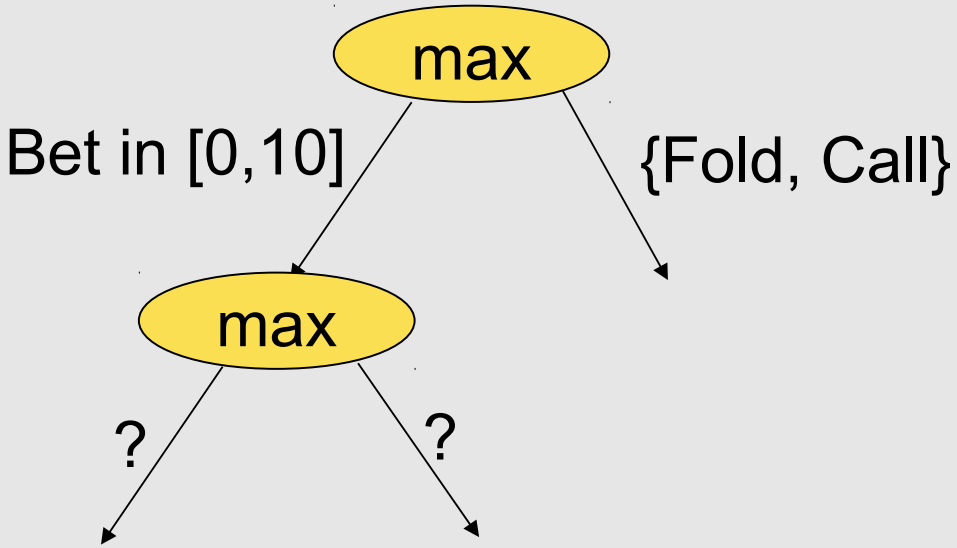


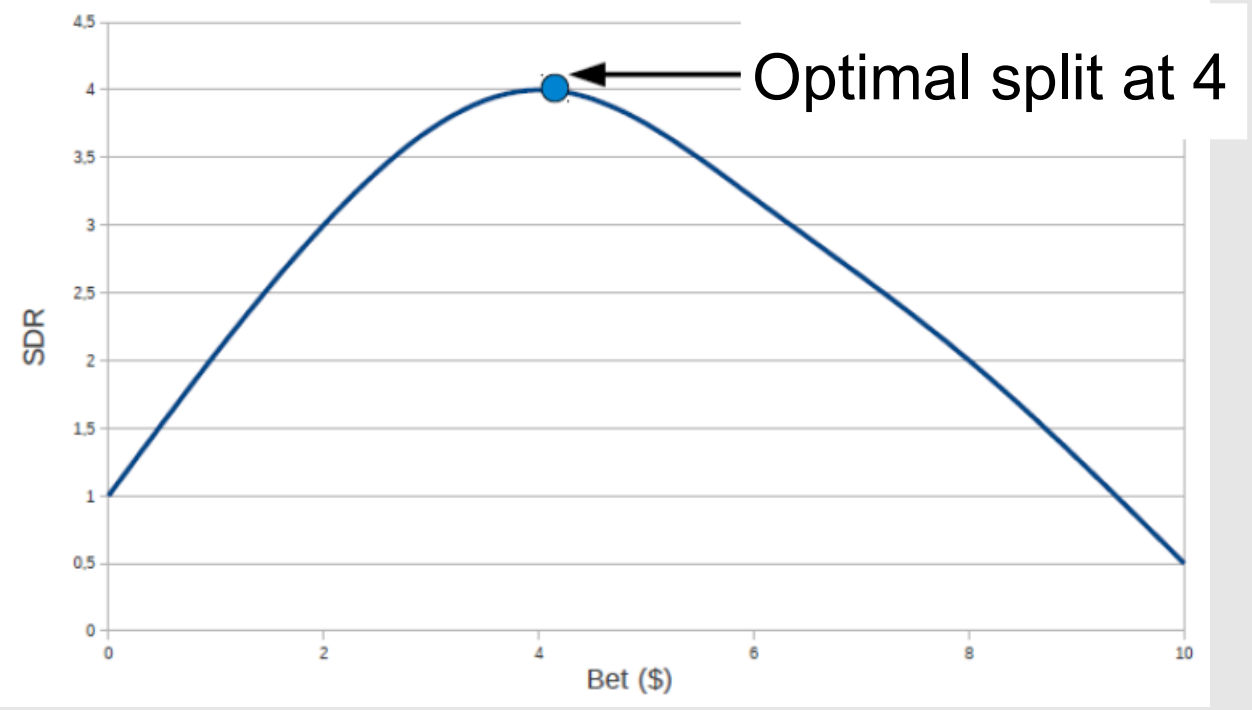
Tree learning search





Tree learning search





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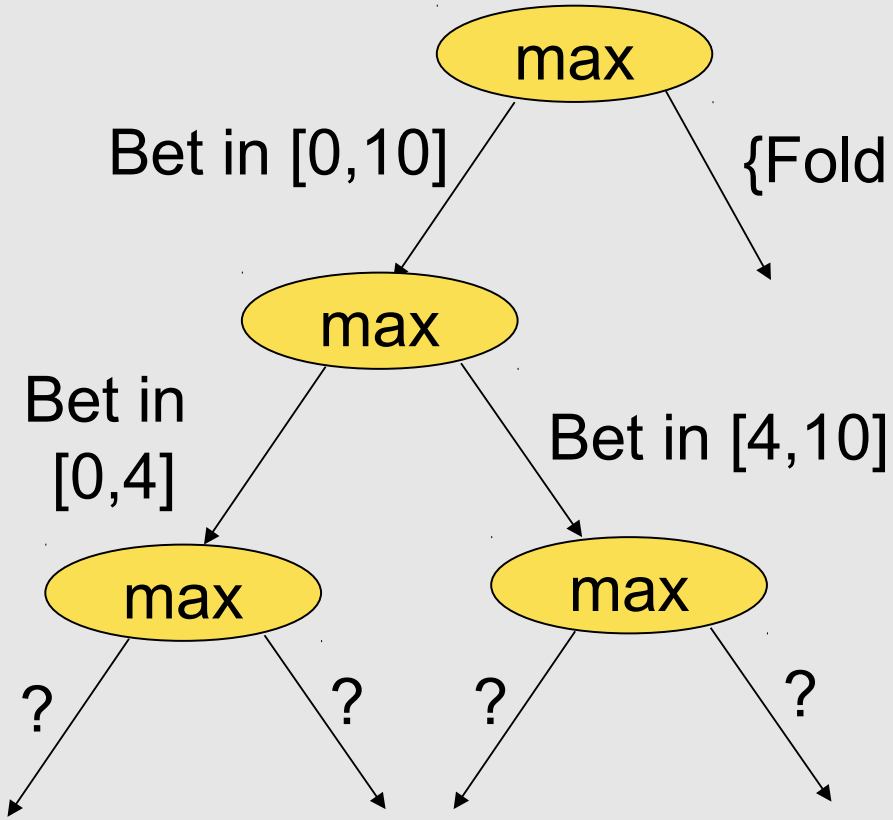

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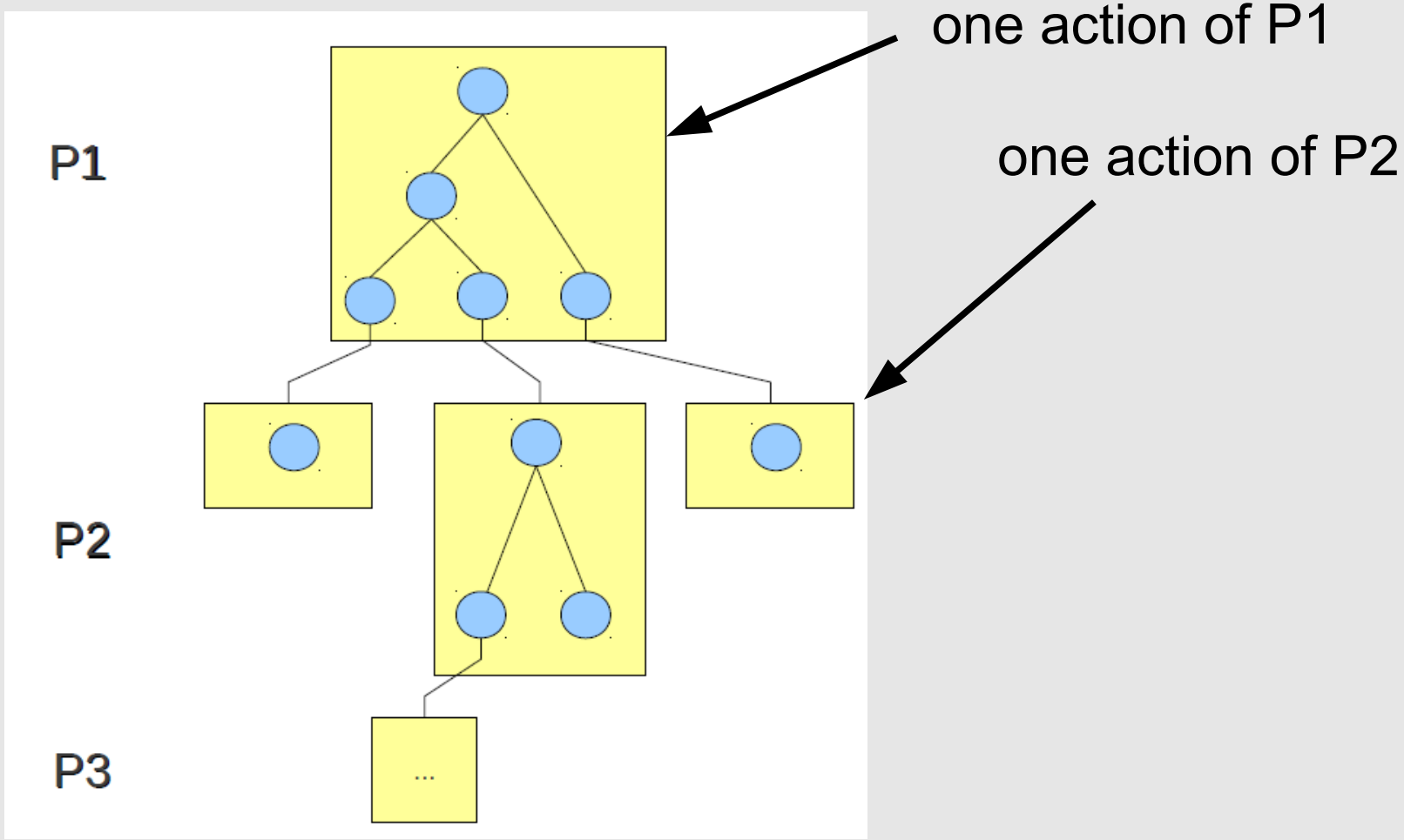
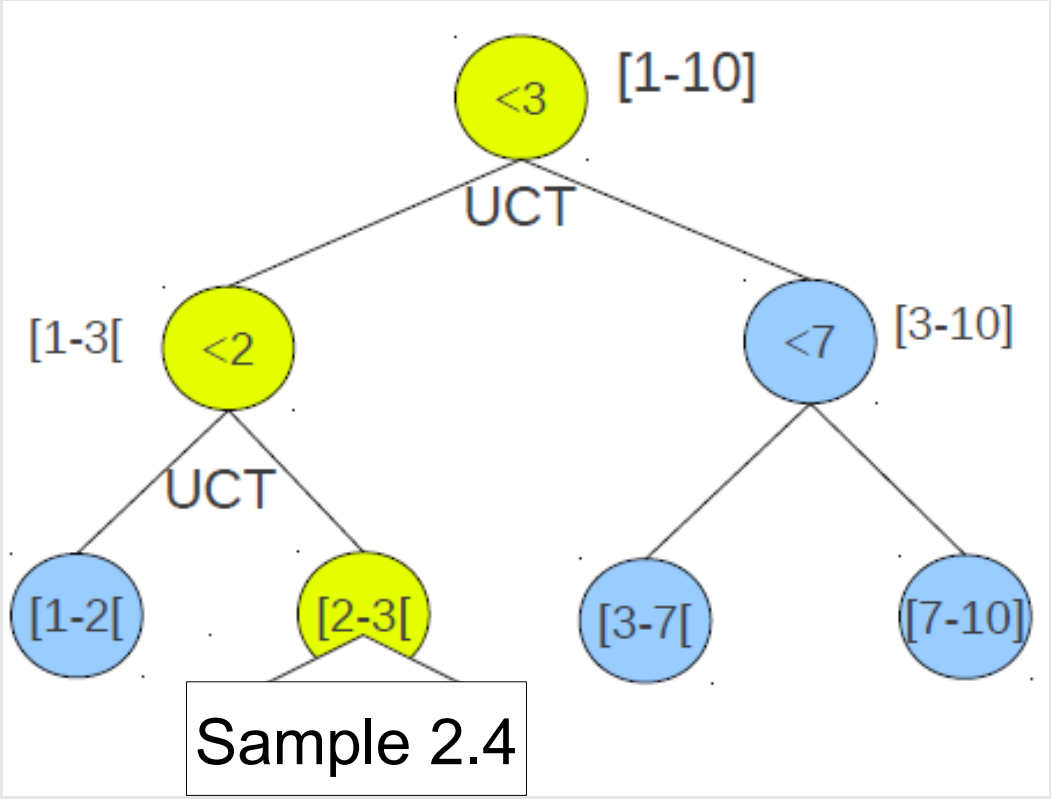
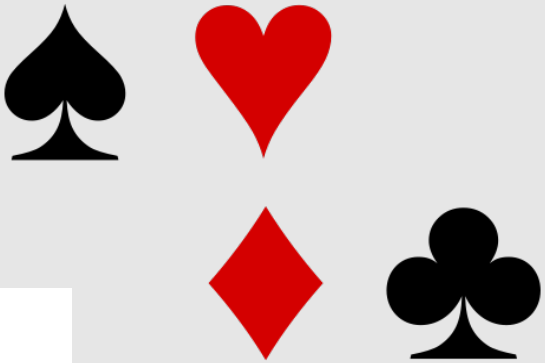


Selection Phase



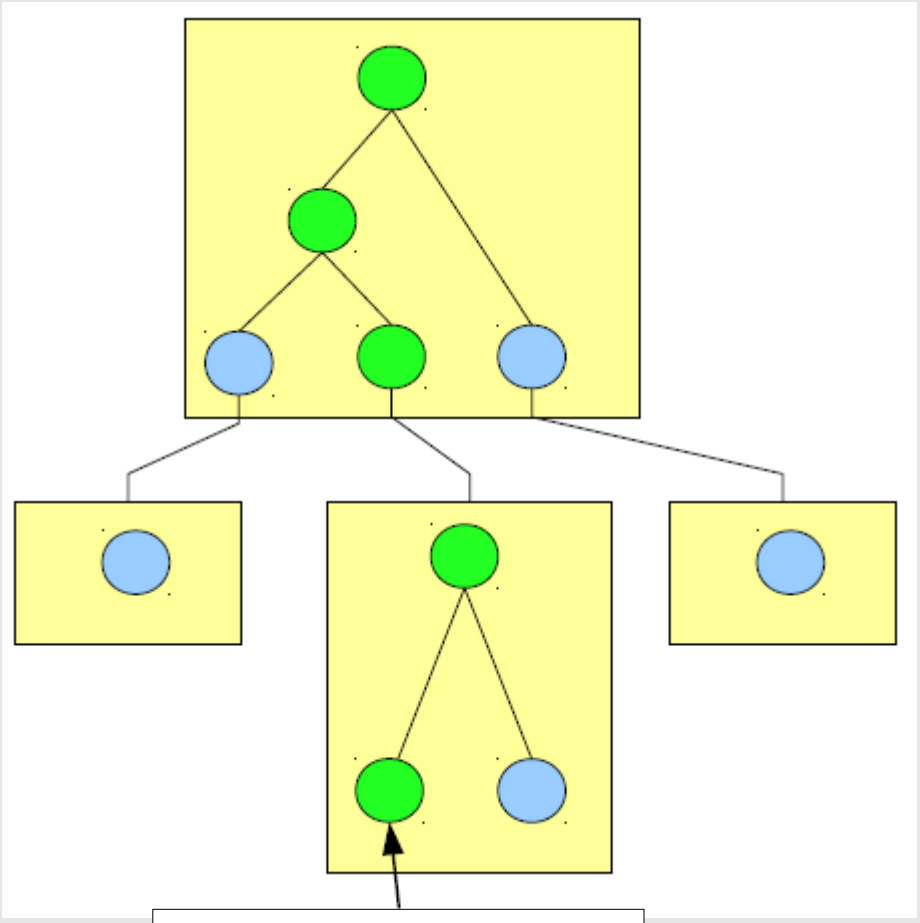
Each node has EV estimate, which generalizes over actions

Expansion

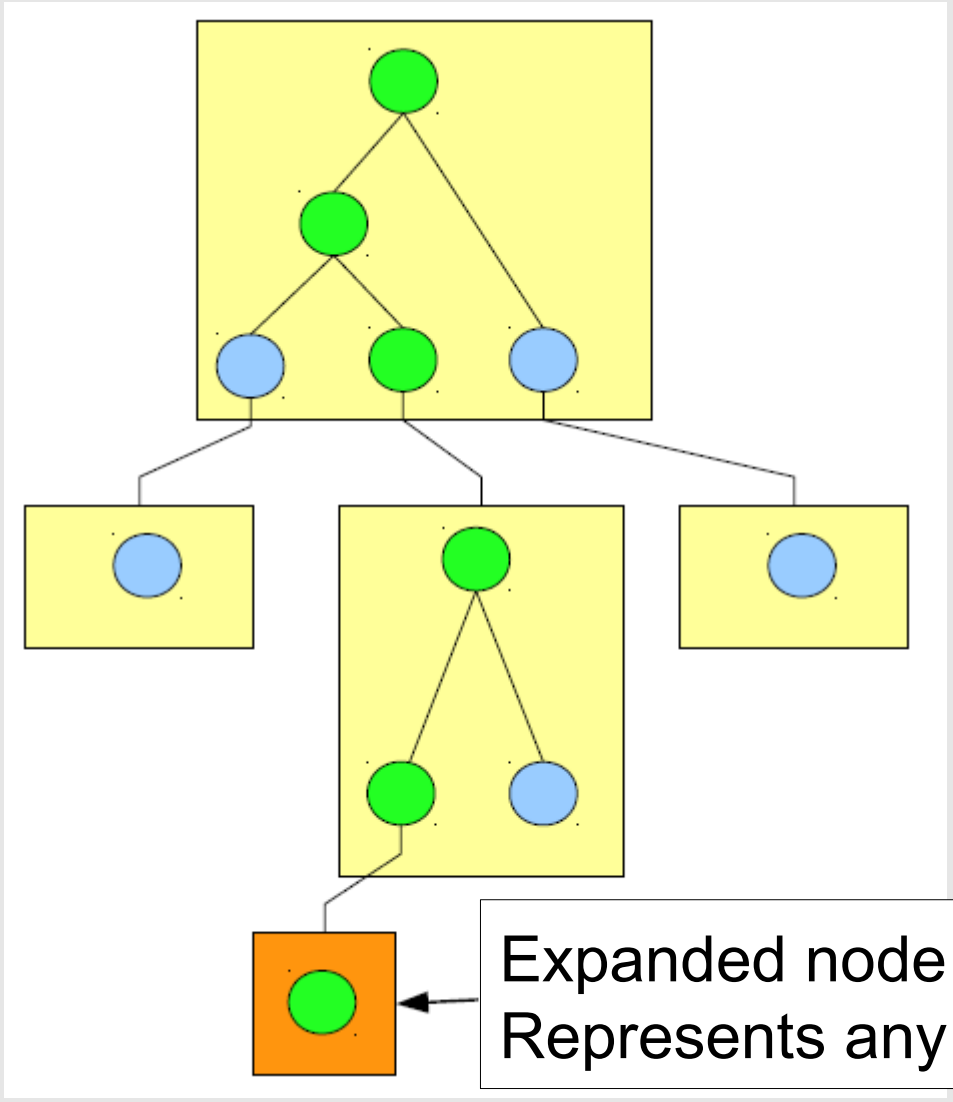


P1

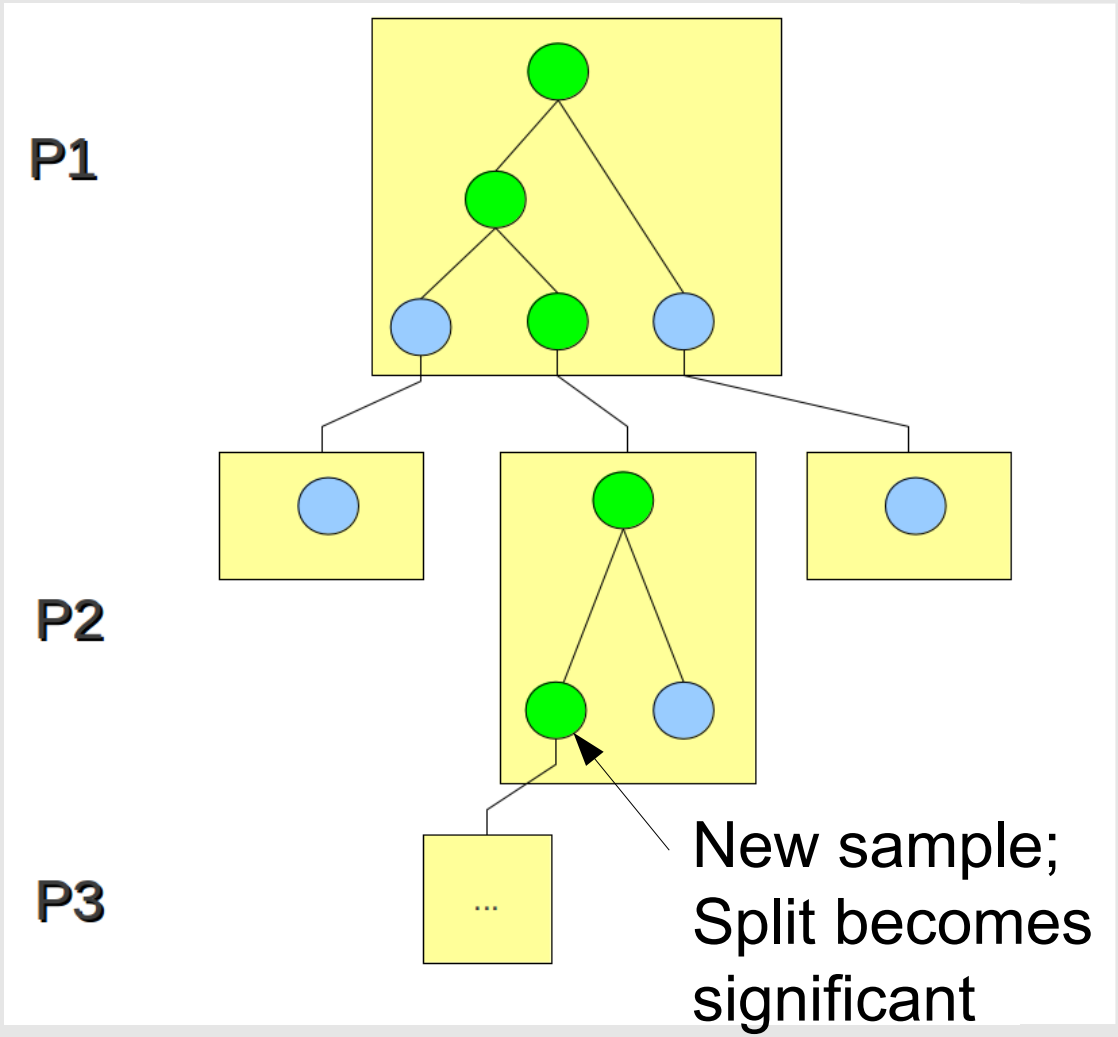
P2



Expansion



Backpropagation



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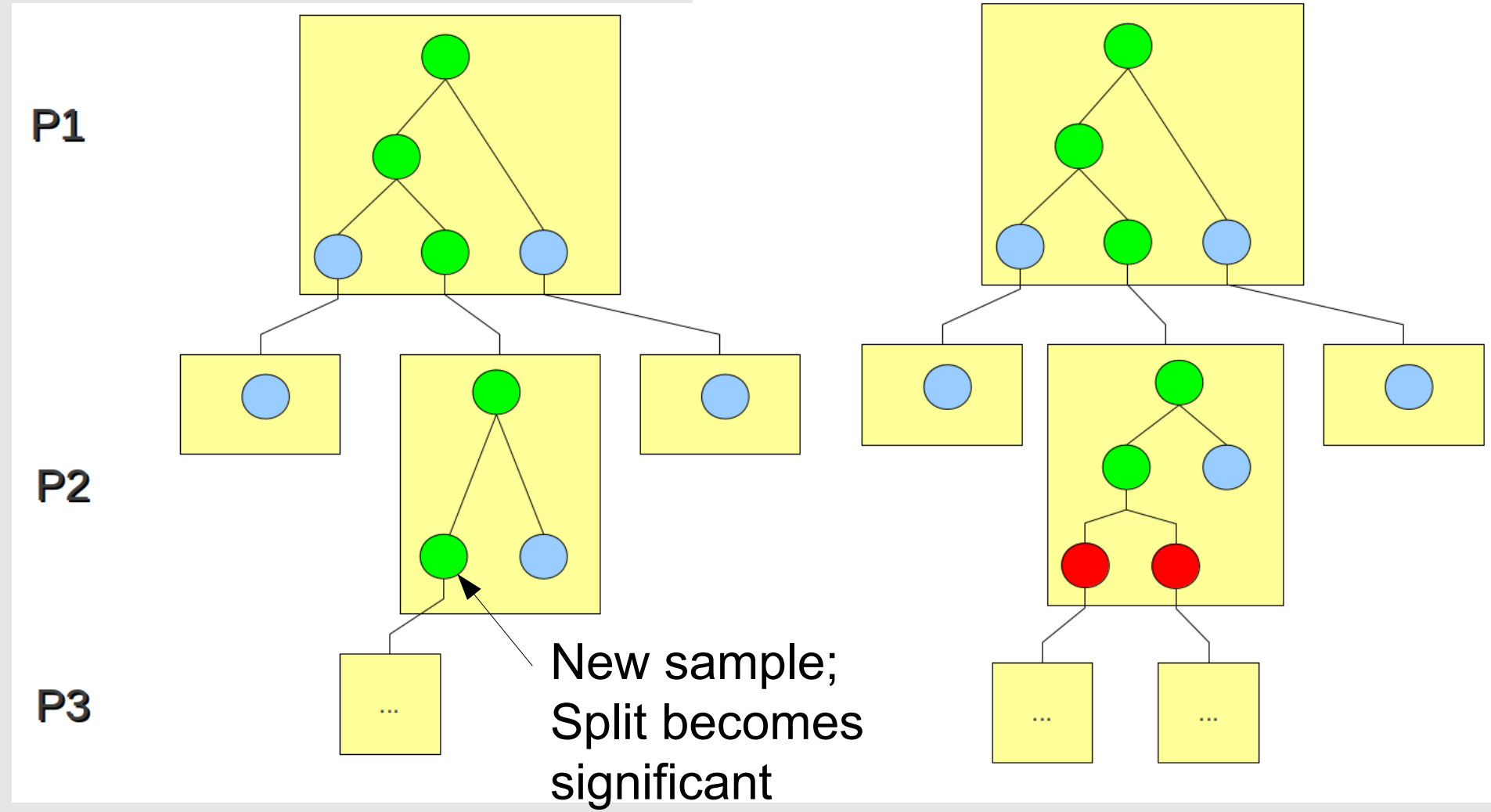

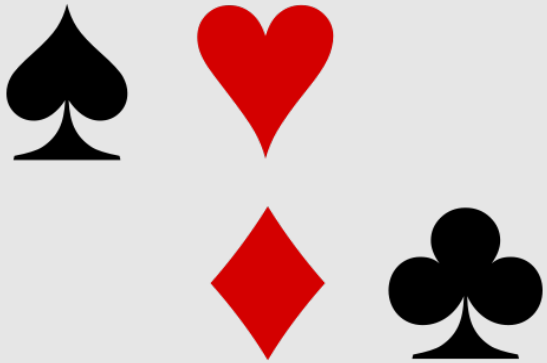

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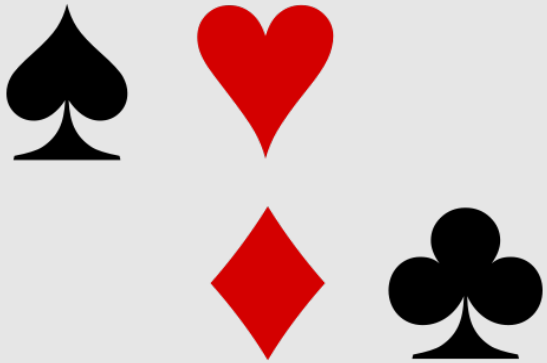

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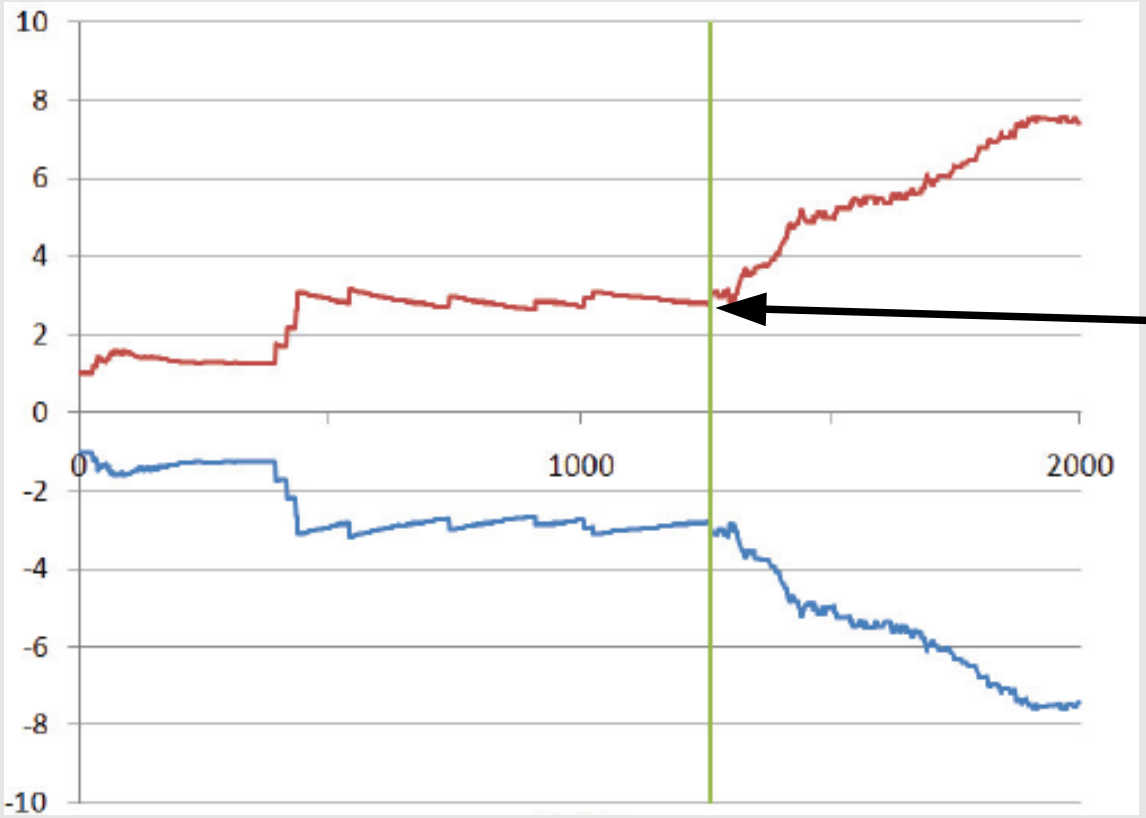
Online learning of opponent model

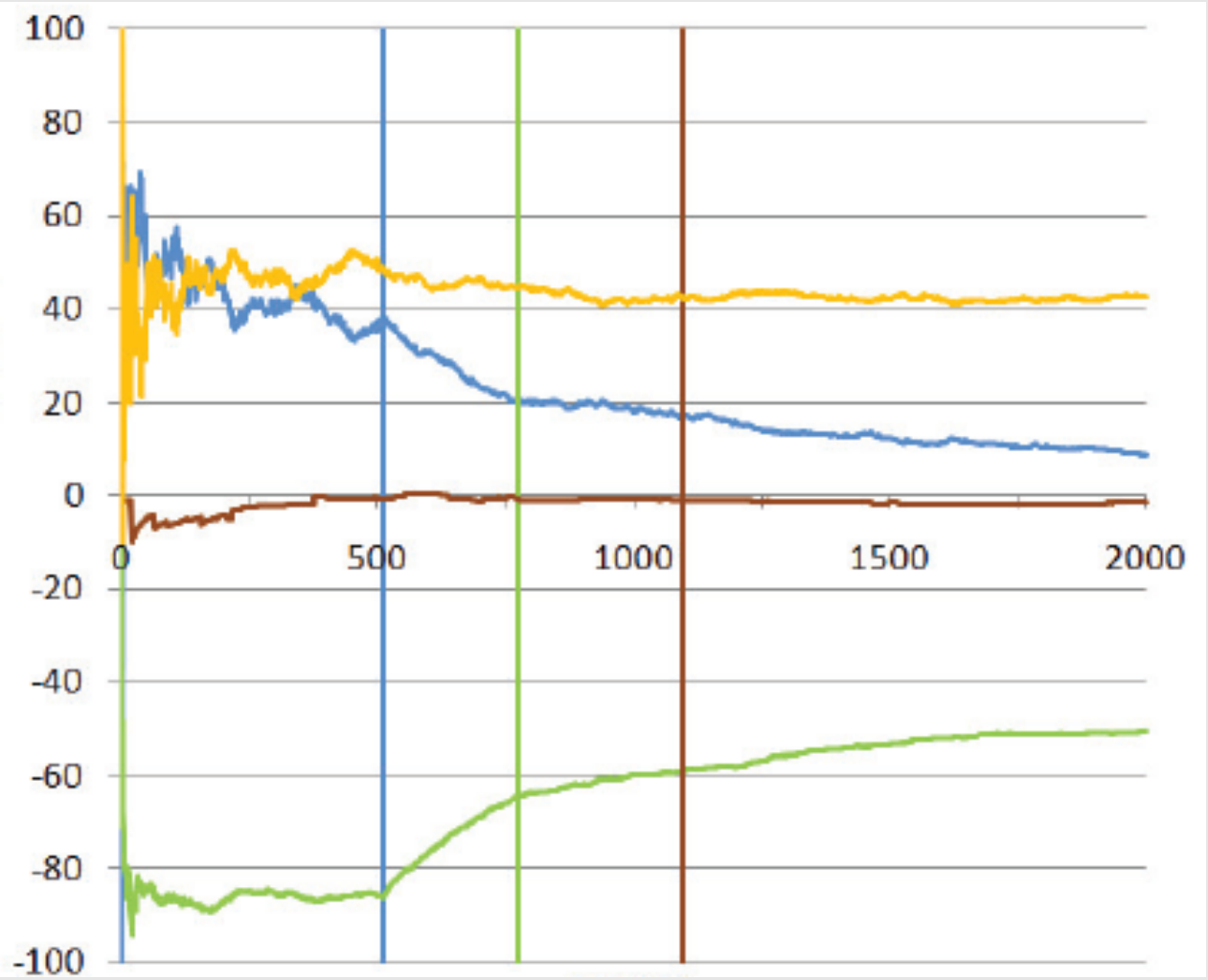
- Start from (safe) model of general opponent
- Exploit weaknesses of specific opponent



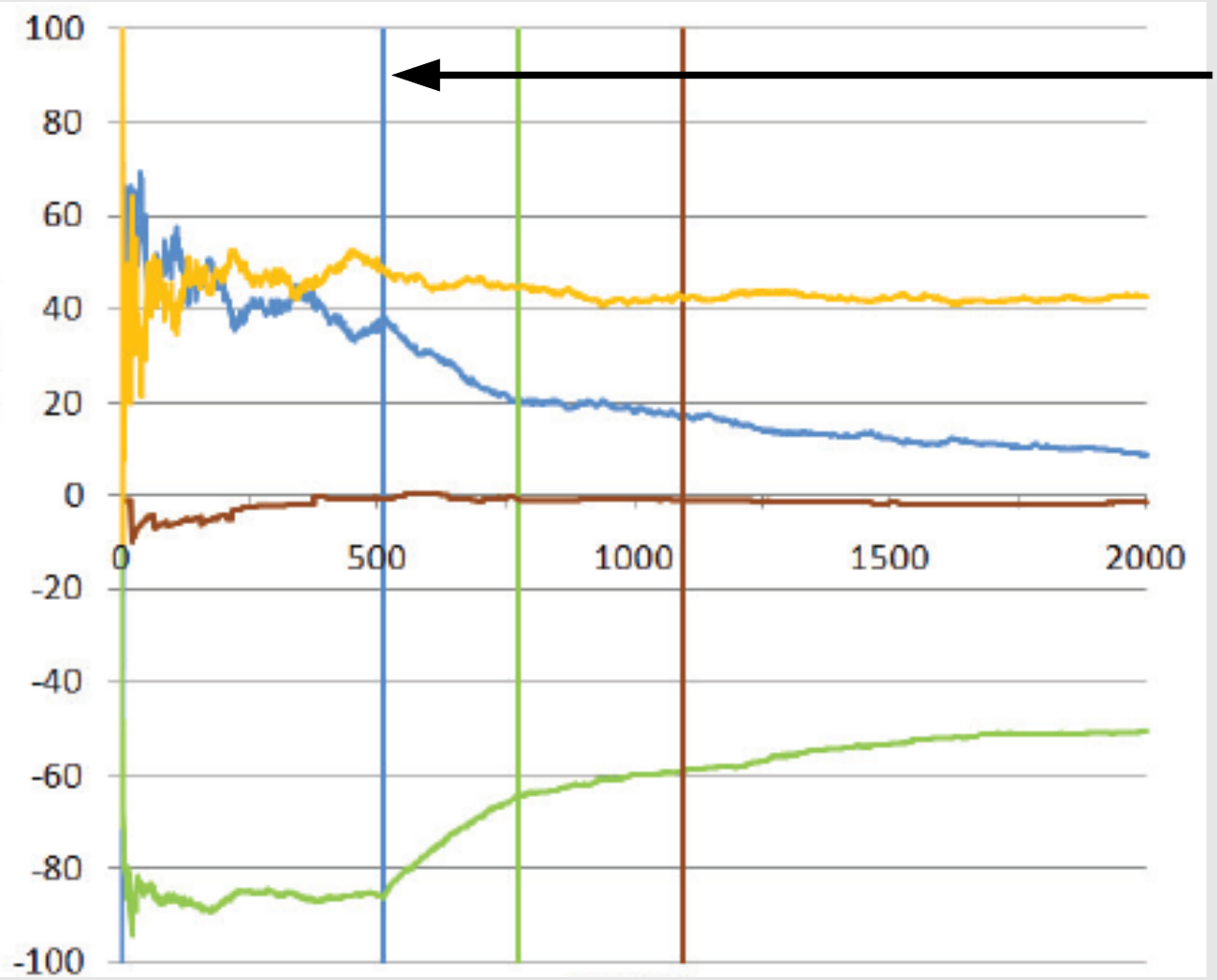
Start to learn model of specific opponent

(exploration of opponent behavior)

Multi-agent interaction

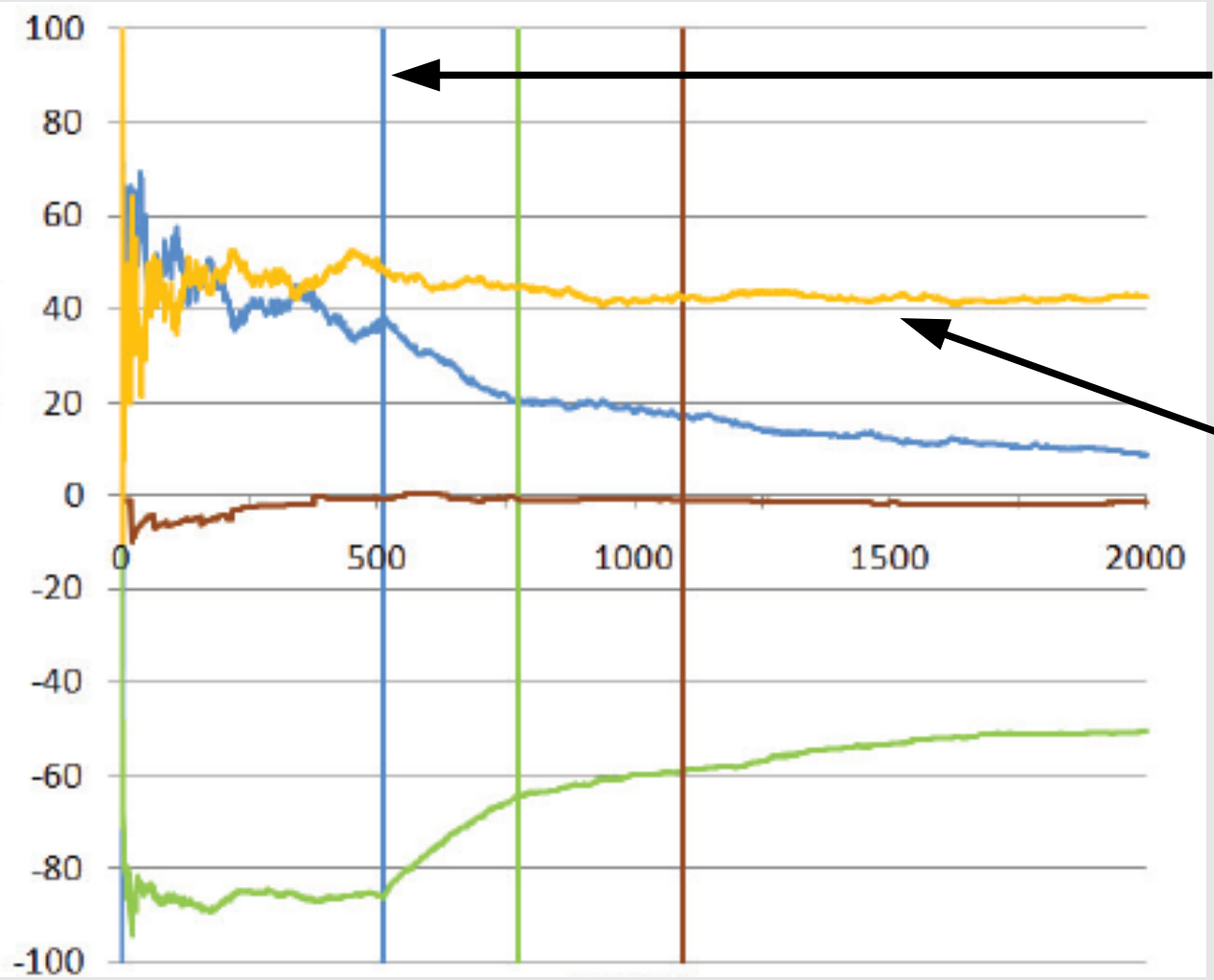


Multi-agent interaction



Yellow learns model for Blue and changes strategy

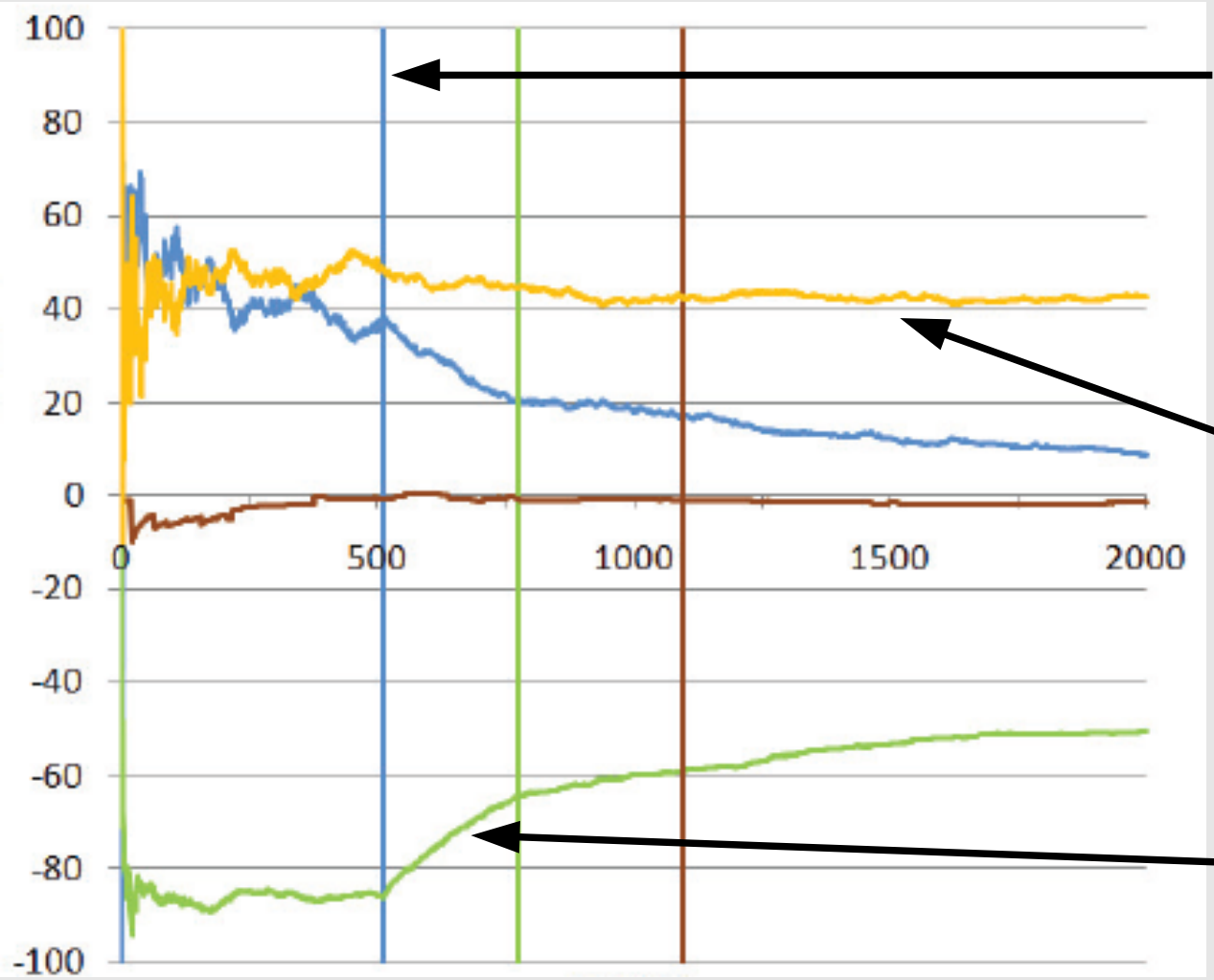
Multi-agent interaction



Yellow learns model for Blue and changes strategy

Yellow doesn't profit!

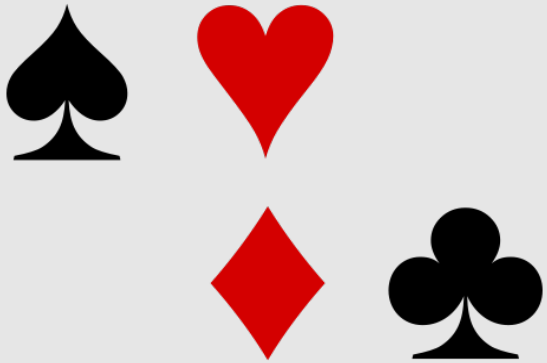
Multi-agent interaction



Yellow learns model for Blue and changes strategy

Yellow doesn't profit!

Green profits without changing strategy!!



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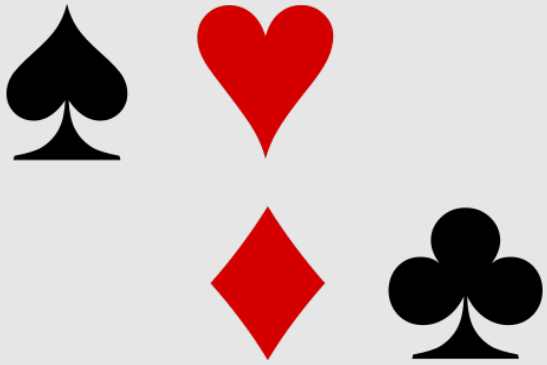
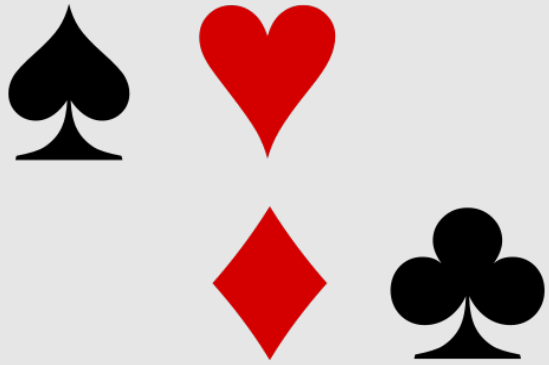
Concept drift



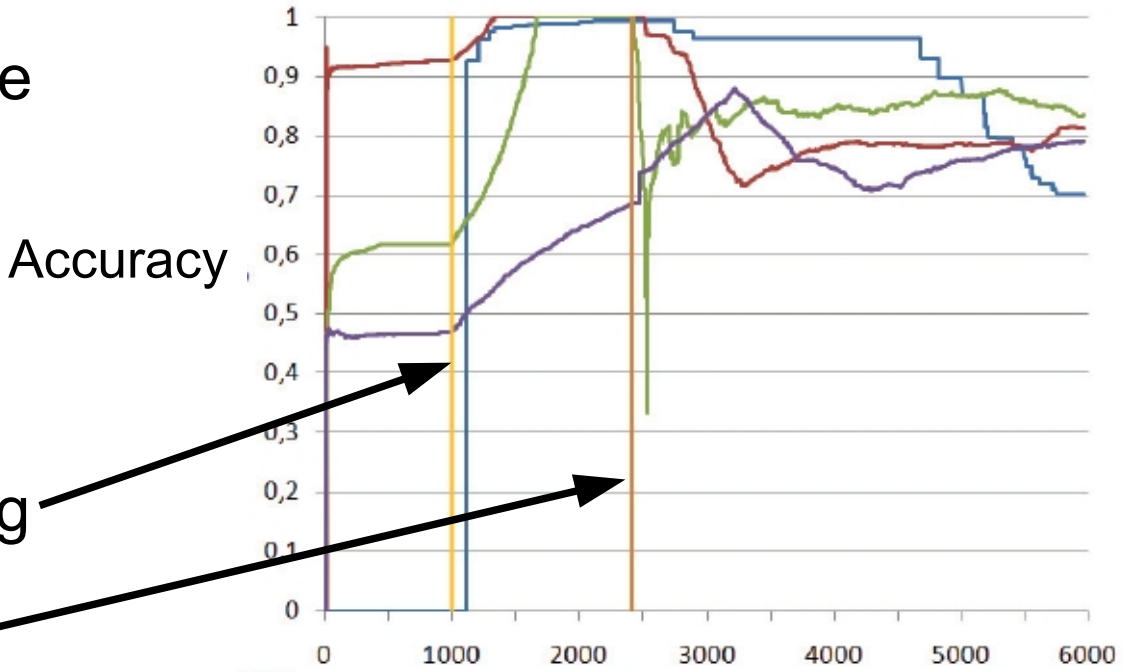
- While learning from a stream, the training examples in the stream change
  - In opponent model: changing strategy
- *"**Changing gears** is not just about bluffing, it's about changing strategy to achieve a goal."*
- Learning with concept drift
  - ***adapt*** quickly to changes
  - yet ***robust*** to noise
  - (recognize recurrent concepts)

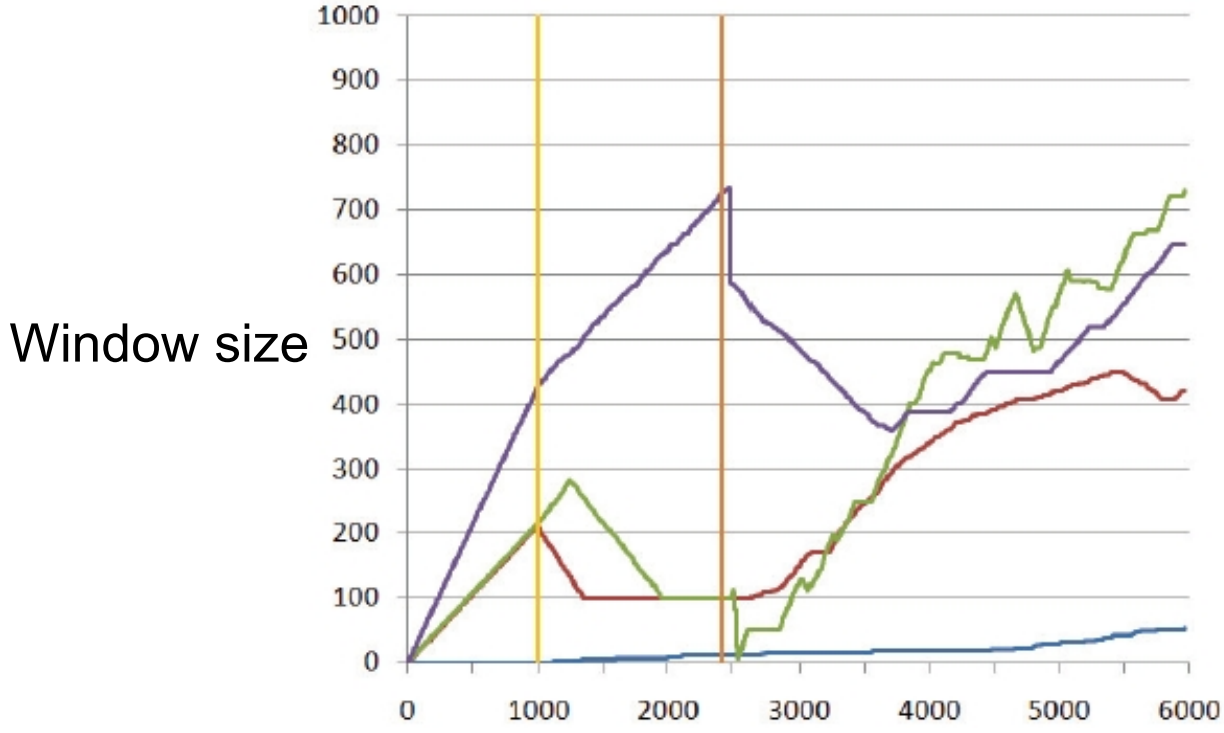
Basic approach to concept drift



- Maintain a window of training examples
  - large enough to learn
  - small enough to adapt quickly
  - without 'old' concepts
- Heuristics to adjust window size
  - based on FLORA2 framework [Widmer92]

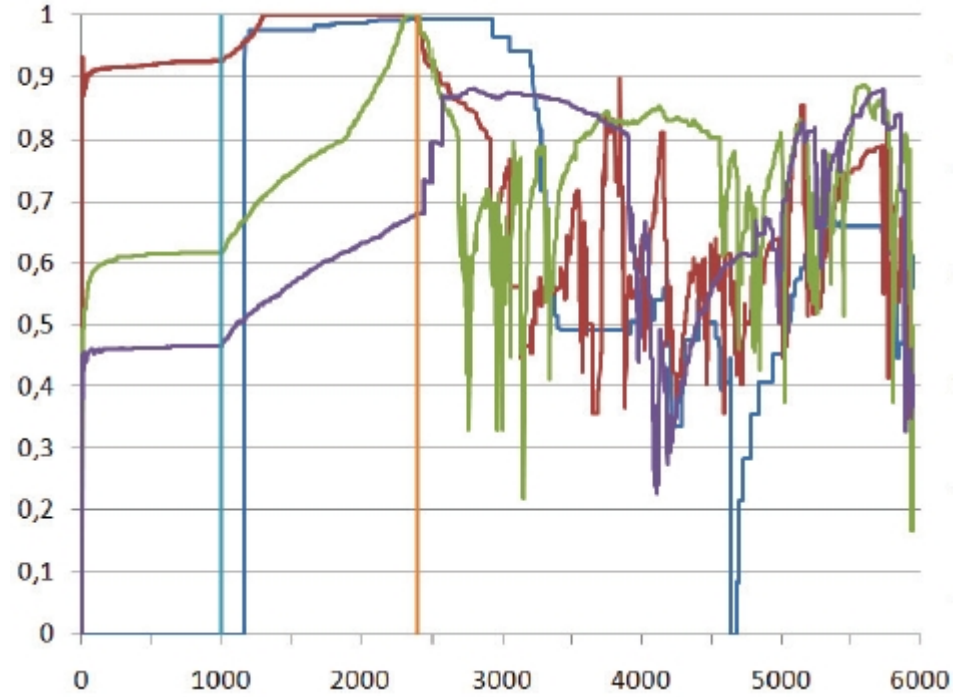
4 components of a single opponent model



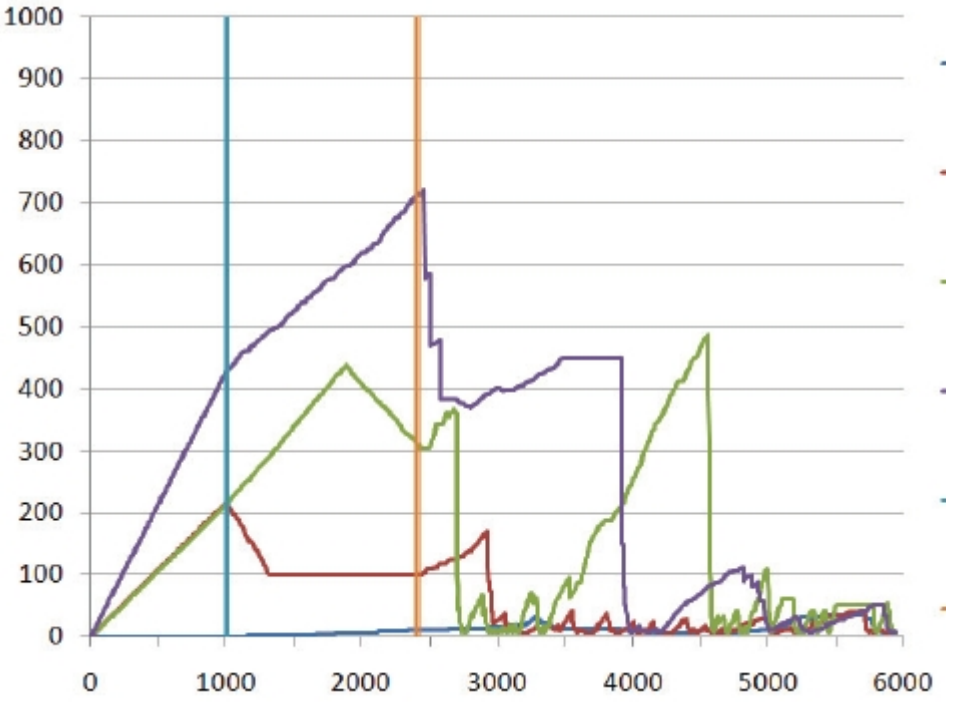


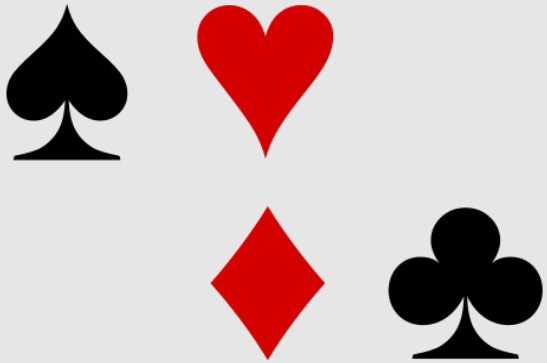
Bad parameters for heuristic

Accuracy



Window size

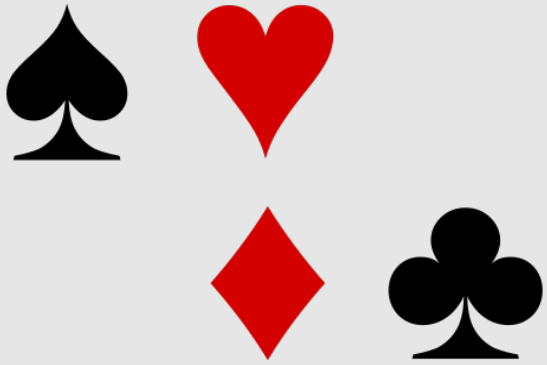




Outline

- Overview approach
  - The Poker game tree
  - Opponent model
  - Monte-Carlo tree search
- Research challenges
  - Search
  - Opponent model
- Conclusion

Conclusions



- First exploitive poker bot for
  - *No-limit* Holdem
  - > 2 players
- Apply in other games
  - backgammon
  - computational pool
  - ...
- Challenge for **MCTS**
  - games with uncertainty
  - continuous action space
- Challenge for **ML**
  - online learning
  - concept drift
  - (relational learning)

Thanks for listening!